Ch. 12 Permutations, Combinations, Probability

Counting the possibilities

1. The international club is planning a trip to Australia and wants to visit Sydney, Melbourne, Brisbane and Alice Springs. In thinking about the order they want to visit each city, they wondered how many different trips they could take. How many possibilities are there?

1st	2nd	3rd	4th
S	M	B	A
M	A	S	B
B	A	S	M
.	.	.	.
.	.	.	.

Can we think of a better way to organize the possibilities?

2. Think about the different combinations you can order a Happy Meal at McDonalds. You can order a hamburger, cheeseburger or chicken nuggets. For sides, you can order fries or yogurt. For drinks, you can order soda, juice or milk.

How many possible combinations can be created?

Note the difference between this and the above example. In the first, we had ***dependent*** events that changed the possibilities depending on our choice where the happy meal choices are all ***independent.***

3. The investment club is electing their officers for next year. They have 12 juniors and 20 other members. They wish to elect a president and a vice president (both which must be juniors) as well as a secretary and treasurer (which can be any member). How many sets of officers are possible?

Principle of Counting: When we are counting the different ways r tasks can be accomplished, we simply multiply the number of possible ways the first task can be accomplished m_1 by the number of possible ways the second task can be accomplished m_2, and so on until we reach the rth task which can be accomplished in m_r ways.

The number of possible ways the tasks can be accomplished together is the product $m_1 m_2 \cdots m_r$

4. Going back to the Australia trip example, what would the possibilities look like if instead, the club was planning a trip to 15 different cities. How would we represent the number of ways they could make that trip?

The shorthand way of representing this expression is called a ***factorial***.

$$n! = n(n-1)(n-2)(n-3) \cdots\cdot 3 \cdot 2 \cdot 1$$

Examples:

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

As a rule, it is convenient to define $0! = 1$

Permutations

A ***permutation*** of *n* different elements is an ordering of the elements such that each element has a specific place in the order (ie, one element is first, another is second, etc.).

The number of permutations of *n* elements is $n!$

5. There are 20 students in the Drama class who are trying out for the four main parts in the play, each a separate named part. How many possible ways can the director cast those four parts? Can we use the factorial notation?

Example 5 is a permutation as well. However, we aren't observing the order of the entire list of *n* elements. We are observing a subset, or ***r*** elements of the *n* elements.

Notation for a permutation of *n* elements observed *r* at a time is $\boxed{{}_n\mathrm{P}_r}$.

To calculate, ${}_n\mathrm{P}_r = \dfrac{n!}{(n-r)!}$

6. Ten horses are running the Kentucky Derby. How many ways can they finish 1^{st}, 2^{nd} and 3^{rd}?

Note, if we are observing all elements (as we were with the Australia trip, going to four cities), we can still use the same notation, this time with *r* equaling *n*. Remember $0! = 1$.

$${}_4\mathrm{P}_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = 4! = 24$$

Distinguishable Permutations

How many ways can the word LOOK be re-arranged? Using our permutation formula, there are 4 elements, of which we are observing all 4. Therefore, there are 4! ways (or 24) ways it can be written.

However, how many can you distinguish? To help illustrate, write out the 24 combinations with numbers representing each letter and then replace with the letters.

1234	1243	1324	1342	1423	1432	2134	2143	2314	2341	2413	2431
LOOK	LOKO	LOOK	LOKO	LKOO	LKOO	OLOK	OLKO	OOLK	OOKL	OKLO	OKOL
4123	4132	4213	4231	4312	4321	3124	3142	3214	3241	3412	3421
KLOO	KLOO	KOLO	KOOL	KOLO	KOOL	OLOK	OLKO	OOLK	OOKL	OKLO	OKOL

When we have indistinguishable elements, we use the following formula to represent the ***distinguishable permutations.***

${}_nP_n = \dfrac{n!}{n_1!n_2!n_3!\cdots}$ where n_1 represents the number of times the first element is repeated, n_2 the second, etc.

How many distinguishable ways can you re-write CANNON?

Homework Assignment Number 1

1. A student taking must take 3 courses, in languages, science and math. Her language options are French, Latin, Spanish, German or Chinese. Her science options are Biology, Physics, and Chemistry. Her math options are Pre-Calculus or Statistics. How many different schedules can she make?

2. When taking an 8 question True-False test, how many possible ways can be the test be submitted (assuming you have to answer every question)?

3. In California, each license plate has a number, followed by three letters, followed by three numbers. How many possible license plates can exist?

4. How many three-letter "words" can be made by putting any three letters together?

5. How many three-letter "words" can be made if no letter can be repeated?

6. How many three-letter "words" can be made with an "o" in the middle?

7. How many three-letter "words" can be made with an "o" in the middle if no letter can be repeated?

8. How many three-letter "words" can be made with a "p" somewhere in the word if no letter can be repeated?

9. If you have a locker combination with 45 possible numbers and 3 turns required, assuming you won't have the same number repeat in your combination, how many combinations are possible?

10. A club has 10 members and is electing four members as president, vice president, secretary and treasurer. How many different officer sets are possible?

11. The Pingry swim team has its best three swimmers going up against DePaul's top three in the final race of the tournament. Coach Reichle is more concerned about the possible finish order as a team than the finish order of the actual swimmers. How many different ways can the team finish?

12. How many distinguishable ways can the following words be re-written?

a) ELEMENT b) BANANA c) MISSISSIPPI

Homework Answers

1. 2 x 3 x 5 = 30

2. 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 = 2^8 = 256

3. 10 x 26 x 26 x 26 x 10 x 10 x 10 = 175,760,000

4. 26 x 26 x 26 = 17,576

5. $_{26}P_3 = \frac{26!}{(26-3)!} = \frac{26\cdot 25\cdot 24\cdot 23!}{23!} = 26\cdot 25\cdot 24 = 15,600$

6. 26 x 26 = 676

7. $_{26}P_2 = \frac{25!}{(25-2)!} = \frac{25!}{23!} = 25\cdot 24 = 600$

8. 3 x 25 x 24 = 1800 (think of how many with p in front , p in middle, p in back)

9. $_{45}P_3 = \frac{45!}{(45-3)!} = \frac{45!}{42!} = 45\cdot 44\cdot 43 = 85,140$

10. $_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 10\cdot 9\cdot 8\cdot 7 = 5,040$

11. distinguishable permutation $\frac{6!}{3!3!} = \frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1} = 20$

PPPDDD	PPDPDD	PPDDPD	PPDDDP	PDPPDD
PDPDPD	PDPDDP	PDDPPD	PDDPDP	PDDDPP
DPPPDD	DPPDPD	DPPDDP	DPDPPD	DPDPDP
DPDDPP	DDPPPD	DDPPDP	DDPDPP	DDDPPP

12.

a. $\frac{7!}{3!} = 7\cdot 6\cdot 5\cdot 4 = 840$

b. $\frac{6!}{3!2!} = \frac{6\cdot 5\cdot 4}{2\cdot 1} = 60$

c. $\frac{11!}{4!4!2!} = \frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5}{4\cdot 3\cdot 2\cdot 1\cdot 2\cdot 1} = 34,650$

Combinations

So far, everything we have been counting has been where the order has been important. It mattered in what "place" we selected each element. However, order isn't always important when counting possibilities.

For instance, poker players want to know how many types of hands they could be dealt. If they are going to be dealt 5 cards, they don't care the order in which the cards came, just what cards they were eventually dealt.

1. Kevin is going on a trip to Tahiti and has room in his suitcase for 2 books. He has a total of 5 to select from on his bookshelf. How many different combinations of books can he bring?

Write out the possible permutations first, remembering that he has $\frac{5!}{3!}$, or 20 possibilities. It may help you to label the novels as letters A, B, C, D, E to organize your thinking.

Now say he picked novels A & B. How many ways could he have picked them? What about A & C?

What if instead, he could have fit 3 books, how many ways could he have picked A, B & C?

A ***combination*** is a subset of *r* elements selected out of *n* distinct elements **without regard to order.** The combination is notated by ${}_nC_r$

The number of combinations is given as ${}_nC_r = \frac{{}_nP_r}{r!}$ or ${}_nC_r = \frac{n!}{r!(n-r)!}$

2. The girls basketball team has 15 members on the team. How many different ways can the coach start a team of 5 players?

3. Gina is ordering the special pizza deal at Papa John's where she gets to pick 3 of the 8 possible toppings for her pizza. How many possible combinations of the 3 toppings can she choose?

4. There are six points on a circle. How many different chords can be drawn by connecting any two of the points? How many different triangles can be inscribed in the circle using any three of the points?

Using combinations to determine probabilities

What is the probability of picking the winning ticket out of a hat of 100 tickets?

What is the probability of winning the raffle is you own 3 tickets out of a total of 50?

Probability is the number of "winning" possibilities divided by the number of total possibilities. Using this, we can now think about using this for combinations.

5. If you were to draw two cards from a 52 card deck. What is the probability of drawing two diamonds?

To start off, think about all the possible two card combinations that you could draw from the deck.

Secondly, think about all the possible ways of drawing two diamonds, making this the "winning condition".

Taking this one step forward, what is the probability of drawing two of the same suit out of a 52 card deck.

6. What is the probability of drawing a pair when drawing two cards from a 52 card deck?

7. The track coach is selecting 12 members to go to the track meet, 6 girls and 6 boys. He has 12 boys and 10 girls to choose from. How many different ways can the coach choose the team?

Homework Assignment Number 2

1. At an important business meeting, 10 executives each introduced themselves to each other, shaking hands with each other exec. How many handshakes occurred?

2. A psych study is being formed from out of the AP Psych class. The study needs 5 of the 25 students, how many different ways can the study group be formed?

3. Tiger has a box of 8 numbered white golf balls and a box of 6 numbered yellow golf balls. How many different ways can Tiger take 3 white balls and 3 yellow balls?

4. How many possible ways can a person submit a ticket for the Powerball lottery where you must select 5 balls out of 55 and one "Powerball" out of 42?

5. The U.S. Senate is picking new members to the 9 member Finance committee. There are 20 Republicans and 14 Democrats vying for the seats.
   a. How many ways can it be selected if it is to be made up of 5 Republicans and 4 Democrats?
   b. How many ways can it be selected if it is to be made up of 4 Republicans and 5 Democrats?

6. A basketball team is made up of 3 centers, 5 forwards and 4 guards. How many ways can the coach field a team of 1 center, 2 forwards and 2 guards?

7. Five women and three men each take turns to choose one ticket from the eight tickets in a hat. Three tickets win prizes and the others are valueless.

   a. What is the probability that all three prizes will be won by women?
   b. What is the probability that two women and one man will win prizes?
   c. What is the probability that at least two women will win prizes?

8. If you are dealt a 5-card poker hand, how many ways can you be dealt four of a kind? (It may help to first think about how many ways you can be dealt four Aces). Therefore, what is the probability of getting dealt a four of a kind hand?

Homework Answers

1. $_{10}C_2 = \frac{10!}{2!8!} = \frac{10\cdot 9}{2\cdot 1} = 45$

2. $_{25}C_5 = \frac{25!}{5!20!} = \frac{25\cdot 24\cdot 23\cdot 22\cdot 21}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 53{,}130$

3. $_8C_3 \cdot {}_6C_3 = \frac{8!}{3!5!}\cdot\frac{6!}{3!3!} = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1}\cdot\frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1} = 1{,}120$

4. $_{55}C_5 \cdot {}_{42}C_1 = \frac{55!}{5!50!}\cdot\frac{42!}{1!41!} = \frac{55\cdot 54\cdot 53\cdot 52\cdot 51}{5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot 42 = 146{,}107{,}962$

5a. $_{20}C_5 \cdot {}_{14}C_4 = \frac{20!}{5!15!}\cdot\frac{14!}{4!10!} = \frac{20\cdot 19\cdot 18\cdot 17\cdot 16}{5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{14\cdot 13\cdot 12\cdot 11}{4\cdot 3\cdot 2\cdot 1} = 15{,}519{,}504$

5b. $_{20}C_4 \cdot {}_{14}C_5 = \frac{20!}{4!16!}\cdot\frac{14!}{5!9!} = \frac{20\cdot 19\cdot 18\cdot 17}{4\cdot 3\cdot 2\cdot 1}\cdot\frac{14\cdot 13\cdot 12\cdot 11\cdot 10}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 9{,}699{,}690$

6. $_3C_1 \cdot {}_5C_2 \cdot {}_4C_2 = \frac{3!}{1!2!}\cdot\frac{5!}{2!3!}\cdot\frac{4!}{2!2!} = 3\cdot\frac{5\cdot 4}{2}\cdot\frac{4\cdot 3}{2} = 180$

7. Possible combinations: $_8C_3 = \frac{8!}{3!5!} = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 56$

a) "Winning" combinations: $_5C_3 = \frac{5!}{3!2!} = \frac{5\cdot 4}{2\cdot 1} = 10$ Probability $= \frac{10}{56} = \frac{5}{28}$

b) "Winning" combinations: $_5C_2 \cdot {}_3C_1 = \frac{5!}{2!3!}\cdot\frac{3!}{1!2!} = 30$ Probability $= \frac{30}{56} = \frac{15}{28}$

c) "Winning" combinations: Part a) + Part b) = 40 Probability $= \frac{40}{56} = \frac{5}{7}$

8. "Winning" combinations: $13 \cdot {}_4C_4 \cdot {}_{48}C_1 = 13\cdot\frac{4!}{4!0!}\cdot 48 = 624$

Possible combinations: $_{52}C_5 = \frac{52!}{5!47!} = 2{,}598{,}960$ Probability $= \frac{624}{2598960} = \frac{1}{4165}$

Probabilities

Some data from the 2000 census, rounded to closest thousands:

- total population: 281,422,000
- total 65 years or older: 34,992,000
- men 65 years or older: 14,410,000
- women 65 years or older: 20,582,000

1. What is the probability that a randomly selected citizen is 65 years or older?

2. What is the probability that a randomly selected citizen is a 65 year or older woman?

3. What is the probability that a randomly selected 65 years or older citizen, is a man?

4. What is the probability that a randomly selected citizen is not 65 years or older?

Probability of an Event often notated as *P(E)* where E is the event in question.

$$P(E)=\frac{n(E)}{n(S)}=\frac{\text{number of possible outcomes of event E}}{\text{total number of possible outcomes in sample S}}$$

When two events have no outcomes in common and are the only two events that can possibly occur, they are called **complementary**.

Which two events from the census problem above are complementary?

What must the probability of complementary events add up to?

Multiple possibilities

5. What is the probability of drawing an ace or a red face card out of a deck of 52 cards?

6. What is the probability of drawing a queen or a spade out of a deck of 52 cards?

Probability of two events given one outcome (as in the last two examples) is as follows:

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

Note that for some examples. *P(E* and *F)* is zero. These events are called mutually exclusive, or disjoint.

7. If you roll a die, what is the probability that the number comes up is less than 4 or is even?

Independent events

8. What is the probability of rolling a one on a die AND flipping a coin heads?

Label the events E and F, $P(E) = \frac{1}{6}$ $P(F) = \frac{1}{2}$ What is *P(E* and *F)*?

9. If the chance of rain for the next three days (assume each day is independent) is 20%, what is the probability of it raining all three days?

9b. What is the probability that is doesn't rain any of the three days?

Conditional Probability

While independent events do occur, many times in real life the probability of one event is related to other events that already occurred.

Examples: Probability of the temperature being 90 degrees Fahrenheit tomorrow if its 90 degrees today.

Probability of getting the lead role in a play after you have been called back for a second round.

Probability of drawing an ace out of a deck of cards after you already hold three.

Conditional probability of event F occurring **given event E has occurred** is written as *P(F|E)*.

10a. There are five blue and five black socks in a drawer. You randomly pick out a blue sock. What is the probability that you will pull a second blue sock out of the drawer next?

10b. There are five blue and five black socks in a drawer. You randomly pick out a blue sock. What is the probability of you pulling out two black socks in a row in your 2nd and 3rd pulls?

The probability of two events occurring when the second is dependent on the other is:

$$P(E \text{ and } F) = P(E) \cdot P(F \mid \text{E})$$

note that if *E* and *F* are independent, *P(F|E) = P(F)*

11. A card is drawn from a deck of 52 cards and observed. It is then returned, the deck is shuffled and another card is drawn, what is the probability both cards are aces?

12. A card is drawn from a deck of 52 cards, then another card is drawn. What is the probability that both cards are aces?

13. What is the probability of drawing two diamonds from a deck of 52 cards without returning the 1st card?

We calculated this before, do we have the same answer? Compare it to $\frac{{}_{13}C_2}{{}_{52}C_2}$

Homework Assignment Number 3

For questions (1-6). In the game Scrabble, there are 100 different tiles each with a different letter on it. Each tile also has a number of points assigned to it from 1 – 10. The table below shows how many of each letter and the point value for that letter. There are a total of 100 tiles.

Tile	Number	Points	Tile	Number	Points	Tile	Number	Points
A	9	1	J	1	8	S	4	1
B	2	3	K	1	5	T	6	1
C	2	3	L	4	1	U	4	1
D	4	2	M	2	3	V	2	4
E	12	1	N	6	1	W	2	4
F	2	4	O	8	1	X	1	8
G	3	2	P	2	3	Y	2	4
H	2	4	Q	1	10	Z	1	10
I	9	1	R	6	1	Blank	2	0

1. What is the probability of the first tile pulled being a vowel?

2. What is the probability of the first tile pulled being worth more than 5 points?

3. What is the probability of the first tile pulled being worth either more than 5 points OR a vowel?

4. What is the probability of the first tile pulled being worth either one point OR in the word “RIGHT”?

5. What is the probability of the first tile pulled not being worth either one point OR in the word “RIGHT”?

6. Assume you pulled six vowels for the first six tiles. What is the probability the seventh pull is a vowel?

7. At the local high school, 28% of the students are juniors, 14% take drama and 45% take the bus to school. If you assume these events are independent, what is the probability that a randomly selected student is a junior who takes drama and the bus to school? Any reason you might think independence should be challenged?

8. If you play a color (red or black) on a roulette wheel, you have a 9/19 chance of winning (47.37%). What is the probability of you winning three plays on a color in a row? What is the probability of you losing three plays on a color in a row?

9. You have one raffle ticket out of 50 tickets in a drawing. There are three prizes, each being pulled in a row, with no possibility of multiple wins for one ticket. What is the probability that you will win the 2^{nd} prize? What about the 3^{rd} prize? What about winning any prize? What about not winning any prize?

Homework Answers

1. $\frac{(9+12+9+8+4)}{100}=\frac{42}{100}=\frac{21}{50}$ or 0.42 or 42%

2. $\frac{4}{100}=\frac{1}{25}$ or .04 or 4%

3. $\frac{42}{100}+\frac{4}{100}=\frac{46}{100}=\frac{23}{50}$ or 0.46 or 46%

4. $\frac{68}{100}+\frac{(6+9+3+2+6)}{100}-\frac{(6+9+6)}{100}=\frac{68+26-21}{100}=\frac{73}{100}$ or 0.73 or 73%

5. 1 – 0.73 = 0.27 or 27%

6. $\frac{36}{94}=\frac{18}{47}\approx 0.383$ or 38.3%

7. 0.01764 or 1.764%.... Are juniors more, less or equally likely to take the bus?

8. $\frac{9}{19}\cdot\frac{9}{19}\cdot\frac{9}{19}\approx .1063$ or 10.63%
   $\frac{10}{19}\cdot\frac{10}{19}\cdot\frac{10}{19}\approx .1458$ or 14.58%

9. 2nd prize: $\frac{49}{50}\cdot\frac{1}{49}=\frac{1}{50}$ or 2%   3rd prize: $\frac{49}{50}\cdot\frac{48}{49}\cdot\frac{1}{48}=\frac{1}{50}$ or 2%

   Any prize: $\frac{1}{50}+\frac{1}{50}+\frac{1}{50}=\frac{3}{50}$ or 6%   Complement $1-\frac{3}{50}=\frac{47}{50}$ or 940025

Chapter 12 Review Sheet

1. Prior to 1995, area codes in the U.S. were restricted. Area codes had to be three digits, with the first digit being any number except 0 or 1. The second digit had to be either 0 or 1. The third digit could be any number 0 to 9. How many area codes were possible?

2. A bicycle lock has 4 numbers on it, ranging from 1 to 9. How many different four digit codes can you make? How many different four digit codes can you make if you didn't want to repeat any numbers?

3. There are 30 people that were called for jury duty. How many different ways can the 12 person jury be selected?

4. There are 13 players on the basketball team.
   a) How many different ways can the coach field a team of 5 players?
   b) How many different ways can the coach specifically select a center, a power forward, a small forward, a shooting guard and a point guard?

5. How many distinguishable ways can you rewrite the word ESSENCE?

6. There are fourteen teams in the American League. How many ways can 6 teams make and be ranked for the playoffs?

7. There are 38 numbers on a roulette wheel: 1-36, 0 and 00. What is the probability of the number 17 coming up twice in a row? What is the probability of the number 17 coming up and then an odd number coming up?

8. Given a typical deck of 52 cards, what is the probability of drawing:
   a) Either a red face card or a diamond when drawing one card.
   b) Two aces (or more) when drawing five cards.
   c) Any three of a kind when drawing three cards.
   d) A flush (ANY five cards of the same suit) when drawing five cards.
   e) Anything that's NOT a flush when drawing five cards.

9. There are five blue socks, three red socks and two green socks in a drawer. What is the probability of:
   a) randomly picking out a red sock.
   b) randomly picking out a pair of green socks with two picks
   c) randomly picking out a red sock and then a blue sock
   d) randomly picking out a red and a blue sock with two picks

Review Answers

1. $8 \cdot 2 \cdot 10 = 160$

2. $9 \cdot 9 \cdot 9 \cdot 9 = 6,561$

$${}_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 9 \cdot 8 \cdot 7 \cdot 6 = 3,024$$

3. $${}_{30}C_{12} = \frac{30!}{12!18!} = 86,493,225$$

4a. $${}_{13}C_5 = \frac{13!}{5!8!} = 1,287$$

4b. $${}_{13}P_5 = \frac{13!}{8!} = 154,440$$

5. $$\frac{7!}{2!3!} = 420$$

6. $${}_{14}P_6 = \frac{14!}{8!} = 2,162,160$$

7. $$\frac{1}{38} \cdot \frac{1}{38} = .00069 \text{ or } .069\%$$

$$\frac{1}{38} \cdot \frac{18}{38} = .0125 \text{ or } 1.25\%$$

8a. $$\frac{6}{52} + \frac{13}{52} - \frac{3}{52} = \frac{16}{52} = 30.8\%$$

8b. ${}_4C_2 \cdot {}_{50}C_3 = 117,600 / {}_{52}C_5 = 2,598,960 \quad = 4.52\%$

8c. $\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot 13 = .235\%$ or $13 \cdot {}_4C_3 = 52 / {}_{52}C_3 = 22,100 \quad = .235\%$

8d. $\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} \cdot 4 = .198\%$ or $4 \cdot {}_{13}C_5 = 5,148 / {}_{52}C_5 = 2,598,960 \quad = .198\%$

8e. $1 - .198\% = 99.802\%$

9a. $$\frac{3}{10}$$

9b. $$\frac{2}{10} \cdot \frac{1}{9} = \frac{1}{45}$$

9c. Prob Red then Blue = $\frac{3}{10} \cdot \frac{5}{9} = \frac{1}{6}$

9d. Prob Blue then Red = $\frac{5}{10} \cdot \frac{3}{9} = \frac{1}{6}$

9d. Prob one Red and one Blue = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$