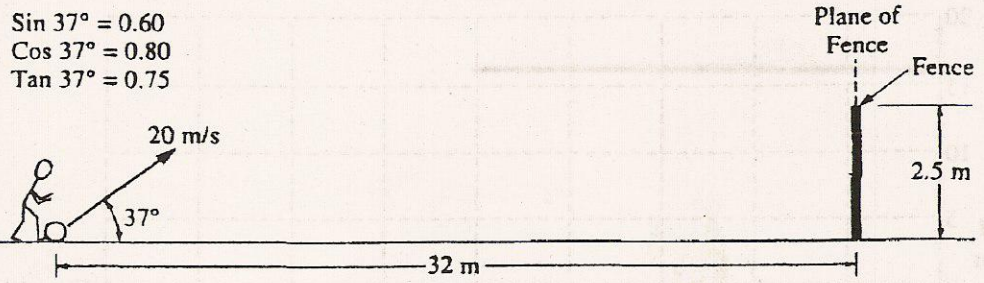
SOLUTIONS

1994-B No. 1

Problem 1



Note: Diagram not drawn to scale.

1. A ball of mass 0.5 kilogram, initially at rest, is kicked directly toward a fence from a point 32 meters away, as shown above. The velocity of the ball as it leaves the kicker's foot is 20 meters per second at an angle of 37° above the horizontal. The top of the fence is 2.5 meters high. The kicker's foot is in contact with the ball for 0.05 second. The ball hits nothing while in flight and air resistance is negligible.

(a) Determine the magnitude of the average net force exerted on the ball during the kick.

(b) Determine the time it takes for the ball to reach the plane of the fence.

(c) Will the ball hit the fence? If so, how far below the top of the fence will it hit? If not, how far above the top of the fence will it pass?

a) $F_{NET} = ma = m\frac{\Delta v}{\Delta t} = 0.5\,kg \cdot \left(\frac{20\frac{m}{s} - 0\frac{m}{s}}{0.05s}\right) = 200N$

b) 20 m/s, 37°, $12\frac{m}{s}$, $16\frac{m}{s}$

$\Delta S_x = \bar{V}_x \Delta t$

$\Delta t = \frac{\Delta S_x}{\bar{V}_x} = \frac{32m}{16 m/s} = \boxed{2 SEC}$

c)

	x	y
ΔS	32m	$\boxed{+4m}$
v_0 ?	16 m/s	+12 m
v_f		$\boxed{-8\frac{m}{s}}$
a	—	$-10 m/s^2$
t	2SEC =	$\boxed{2SEC}$

$\leftarrow \Delta S_y = \bar{V}_y \Delta t = \left(\frac{12 + -8}{2}\right)2 = 4m$

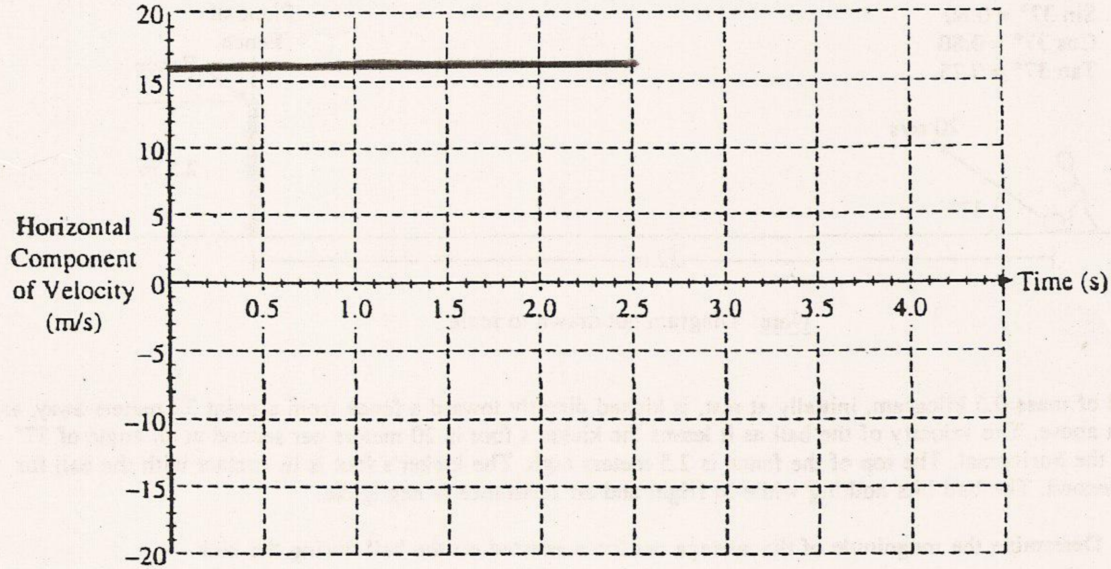
$\leftarrow v_f = v_0 + at$

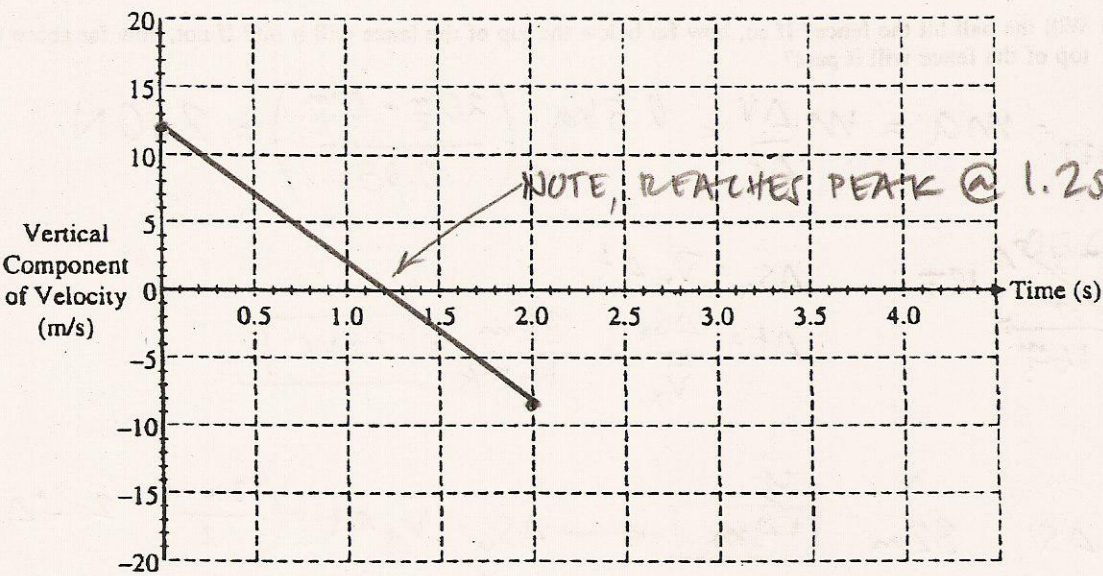
$= +12 + (-10\frac{m}{s^2})(2SEC) = -8 m/s$

THEREFORE CLEARS FENCE BY 4 − 2.5 = 1.5m

Problem 1 (cont.)

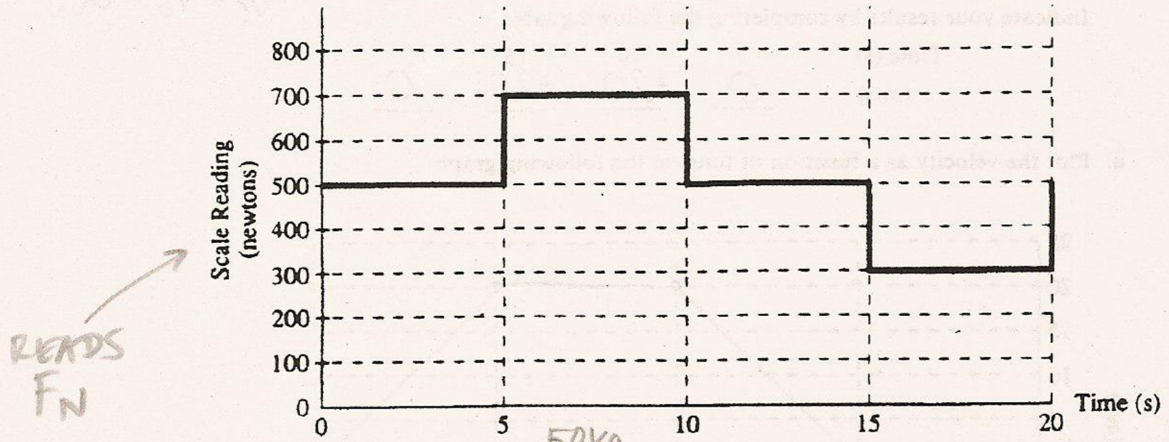
(d) On the axes below, sketch the horizontal and vertical components of the velocity of the ball as functions of time until the ball reaches the plane of the fence.





50

Problem 2



← 50kg

1. A student whose normal weight is 500 newtons stands on a scale in an elevator and records the scale reading as a function of time. The data are shown in the graph above. At time $t = 0$, the elevator is at displacement $x = 0$ with velocity $v = 0$. Assume that the positive directions for displacement, velocity, and acceleration are upward.

   (a) On the diagram to the right, draw and label all of the forces on the student at $t = 8$ seconds.

$F_{NET} = F_N - W$

$= ma$

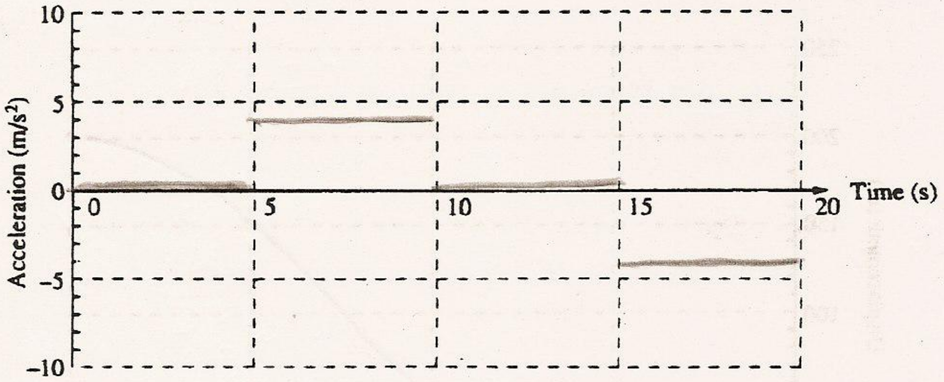
   (b) Calculate the acceleration a of the elevator for each 5-second interval.

$a = \frac{F_N - W}{m}$

   i. Indicate your results by completing the following table.

Time Interval (s)	0–5	5–10	10–15	15–20
a (m/s^2)	0	+4	0	−4

   ii. Plot the acceleration as a function of time on the following graph.



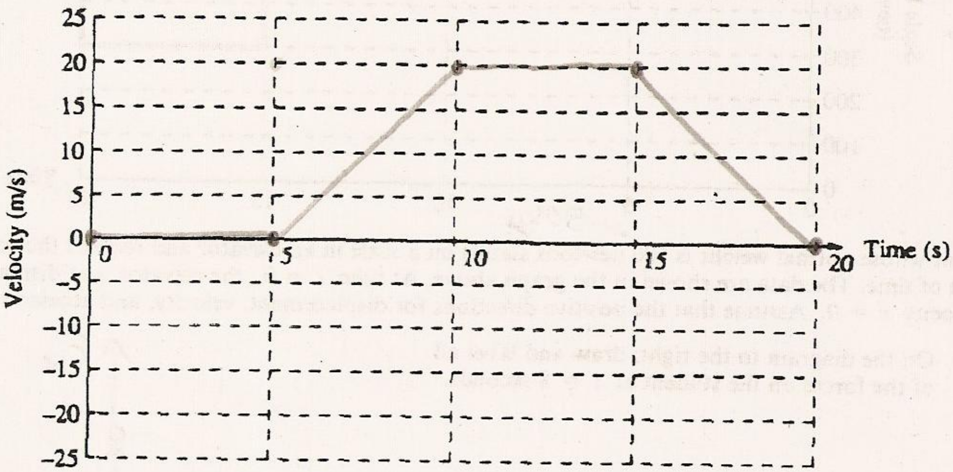
(c) Determine the velocity v of the elevator at the end of each 5-second interval.

$V_f = V_0 + at$

i. Indicate your results by completing the following table.

Time (s)	5	10	15	20
v (m/s)	0	+20	+20	0

ii. Plot the velocity as a function of time on the following graph.



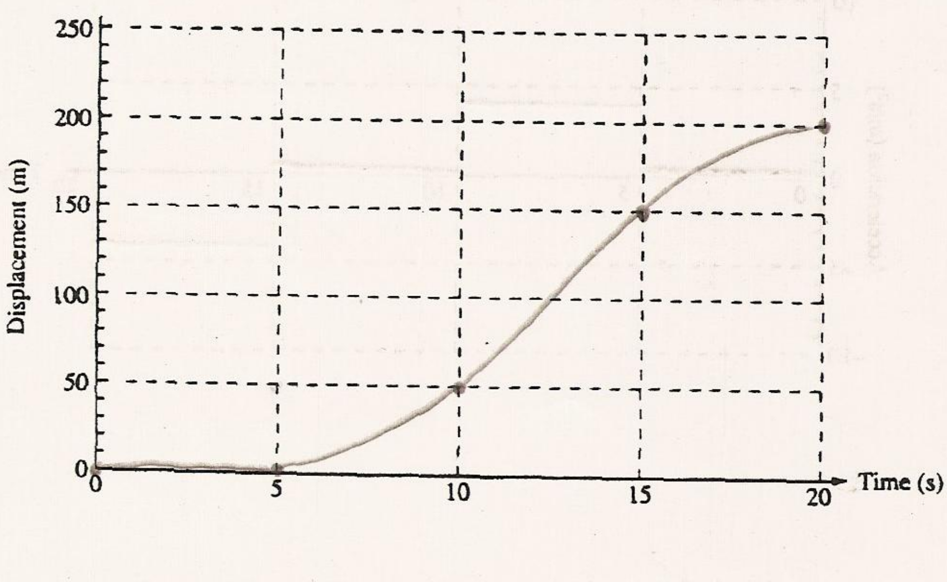
(d) Determine the displacement x of the elevator above the starting point at the end of each 5-second interval.

$\Delta S = \bar{V} \Delta t$

i. Indicate your results by completing the following table.

Time (s)	5	10	15	20
x (m)	0	50	150	200

ii. Plot the displacement as a function of time on the following graph.

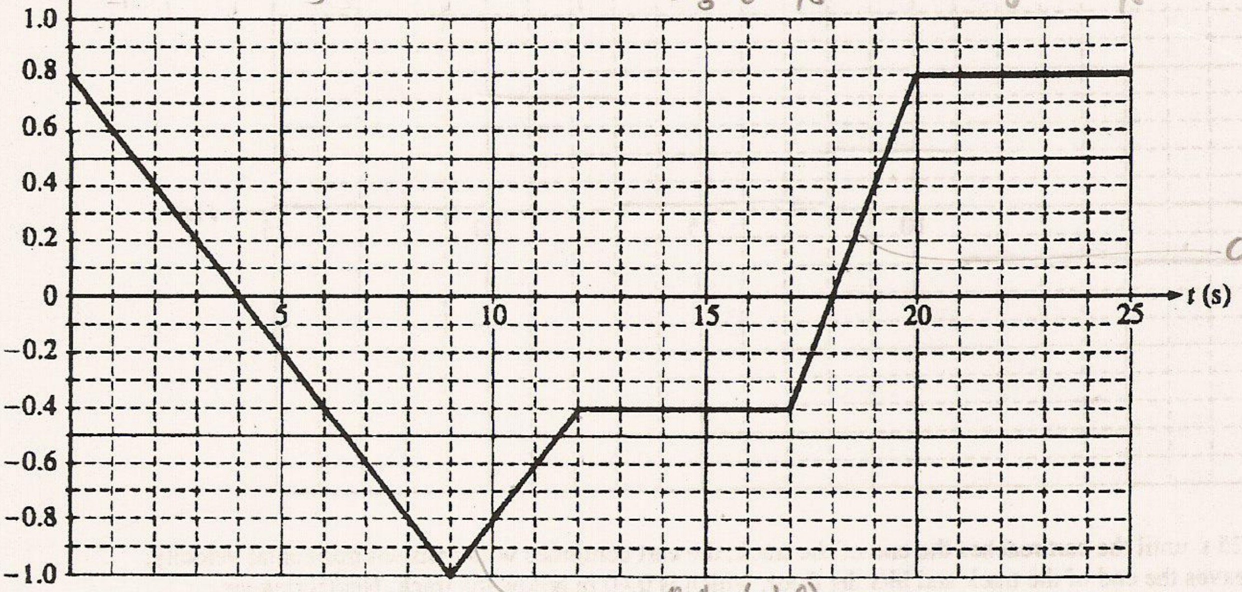


Problem 3

1. (15 points)

A 0.50 kg cart moves on a straight horizontal track. The graph of velocity v_x versus time t for the cart is given below.

$a_1 = \frac{-1.0-(0.8)}{9} = -0.2\,m/s^2$ $\quad a_3 = 0\,m/s^2 \quad a_5 = 0\,m/s^2$



$a_4 = \frac{0.8-(-0.4)}{3\,SEC} = 0.4\,m/s^2$

$a_2 = \frac{-0.4-(-1.0)}{3\,SEC} = 0.2\,m/s^2$

(a) Indicate every time t for which the cart is at rest.

(b) Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing.

(c) Determine the horizontal position x of the cart at $t = 9.0$ s if the cart is located at $x = 2.0$ m at $t = 0$.

a) @ REST @ t = 4, 18 SEC

b) SPEED INCREASING FOR INTERVALS

t = 4 → 9 SEC

= 17 → 20 SEC

c) $\Delta S_1 = \bar{V}_1 \Delta t_1 = 0.4\frac{m}{s} \cdot 4\,SEC = +1.6\,m$

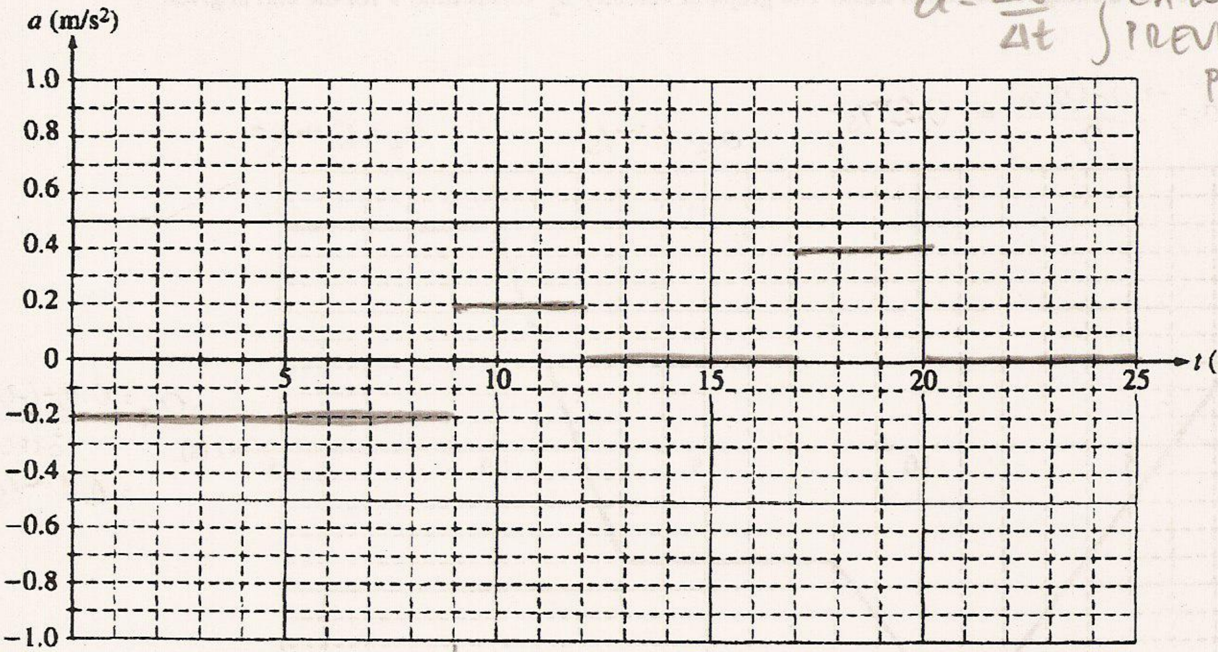
$\Delta S_2 = \bar{V}_2 \Delta t_2 = -0.5\frac{m}{s} \cdot 5 = -2.5\,m$

$S_f = S_0 + \Delta S_1 + \Delta S_2 = 2.0\,m + 1.6\,m - 2.5\,m = +1.1\,m$

Problem 3 (cont.)

(d) On the axes below, sketch the acceleration a *versus* time t graph for the motion of the cart from $t = 0$ to $t = 25$ s.

$a = \frac{\Delta v}{\Delta t}$ } CALC'S ON PREVIOUS PAGE



(e) From $t = 25$ s until the cart reaches the end of the track, the cart continues with constant horizontal velocity. The cart leaves the end of the track and hits the floor, which is 0.40 m below the track. Neglecting air resistance, determine each of the following.

0.8 m/s

i. The time from when the cart leaves the track until it first hits the floor 0.283s

ii. The horizontal distance from the end of the track to the point at which the cart first hits the floor 0.23 m

iii. The kinetic energy of the cart immediately before it hits the floor

0.8 m/s

−2.83 m/s

v = 2.94 m/s

$\boxed{KE = \frac{1}{2}(0.5\,kg)(2.94)^2 = 2.16\,J}$

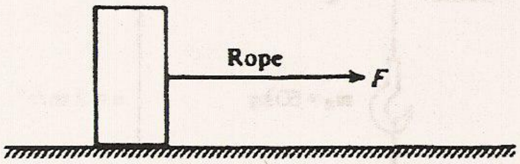
	x	y
Δs	$\boxed{0.23\,m}$	$-0.40\,m$
v_0	0.8 m/s	0 m/s
v_f	0.8 m/s	$\boxed{-2.83}$
a	—	$-10\,m/s^2$
t	$\boxed{0.283}$ =	$\boxed{0.283}$

$v_f^2 = \cancel{v_0^2}^{\,0} + 2a\Delta s$
$= 2(-10)(-0.4) \rightarrow v_f = -2.83\,m/s$ DOWN

$t = \frac{v_f - v_0}{a} = \frac{-2.83}{-10}$

$\Delta s_x = 0.8\,\frac{m}{s} \cdot 0.283\,s = 0.226\,m$
(0.23 m)

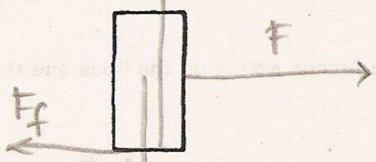
Problem 4



1. A box of uniform density weighing 100 newtons moves in a straight line with constant speed along a horizontal surface. The coefficient of sliding friction is 0.4 and a rope exerts a force F in the direction of motion as shown above.

F_N

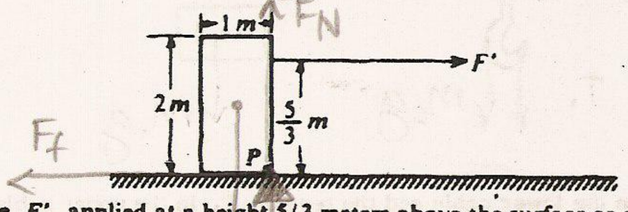
(a) On the diagram below, draw and identify all the forces on the box.



(b) Calculate the force F exerted by the rope that keeps the box moving with constant speed.

Mg

$F = F_f = \mu F_N = \mu Mg = 0.4 \cdot 100N = 40N$



(c) A horizontal force F', applied at a height 5/3 meters above the surface as shown in the diagram above, is just sufficient to cause the box to begin to tip forward about an axis through point P. The box is 1 meter wide and 2 meters high. Calculate the force F'.

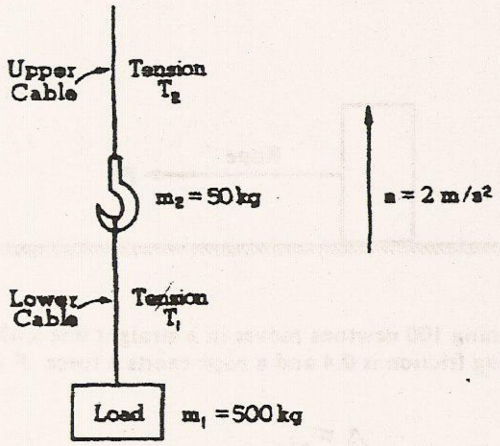
Mg

SOLVE BY τ

$$Mg \cdot d_{\perp w} - F' d_{\perp F'} = 0$$

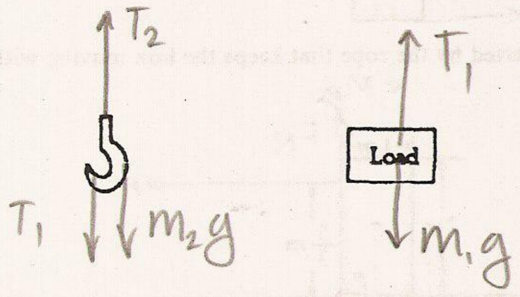
$$F' = \frac{100N \cdot 0.5 m_\perp}{5/3\ m_\perp} = 30N$$

Problem 5



2. A crane is used to hoist a load of mass $m_1 = 500$ kilograms. The load is suspended by a cable from a hook of mass $m_2 = 50$ kilograms, as shown in the diagram above. The load is lifted upward at a constant acceleration of 2 m/s².

(a) On the diagrams below, draw and label the forces acting on the hook and the forces acting on the load as they accelerate upward.



(b) Determine the tension T_1 in the lower cable and the tension T_2 in the upper cable as the hook and load are accelerated upward at 2 m/s². Use g = 10 m/s².

$T_2 - T_1 - m_2 g = m_2 a$ SAME $T_1 - m_1 g = m_1 a$

SUB-IN $T_1 = m_1 a + m_1 g$

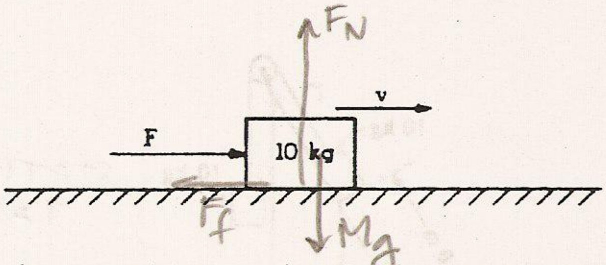
$T_2 - m_1 a - m_1 g - m_2 g = m_2 a$

$T_2 = m_2 a + m_1 a + m_1 g + m_2 g$
$= (50)(2) + 500(2) + 500(10) + 50(10)$

$\boxed{T_2 = 6600\text{N}}$

and $T_1 = T_2 - m_2 a - m_2 g = 6600 - 50(2) - 50(10)$
$\boxed{T_1 = 6000\text{N}}$

Problem 6



1. A 10-kilogram block is pushed along a rough horizontal surface by a constant horizontal force F as shown above. At time t = 0, the velocity v of the block is 6.0 meters per second in the same direction as the force. The coefficient of sliding friction is 0.2. Assume g = 10 meters per second squared.

(a) Calculate the force F necessary to keep the velocity constant.

The force is now changed to a larger constant value F′. The block accelerates so that its kinetic energy increases by 60 joules while it slides a distance of 4.0 meters.

(b) Calculate the force F′.
(c) Calculate the acceleration of the block.

a) $V = \text{CONSTANT} \therefore a = 0 \therefore F = F_f$

$= \mu F_N$

$= \mu Mg = 0.2(10\text{kg})(10\frac{m}{s^2})$

$= \boxed{20\text{N}}$

b) $\Delta KE = W_{BY\ F_{NET}}$

$= F_{NET} \cdot \Delta S_{\parallel}$

$F_{NET} = \frac{60\text{J}}{4\text{m}} = 15\text{ N} \therefore F' - F_f = F_{NET}$

so $F' = F_{NET} + F_f = 15\text{N} + 20\text{N}$

$\boxed{= 35\text{N}}$

c) $a = \frac{F_{NET}}{m} = \frac{15\text{N}}{10\text{kg}} = \boxed{1.5\ \text{m/s}^2}$

CHECK: $KE_0 = \frac{1}{2}(10)(6)^2 = 180\text{J}$

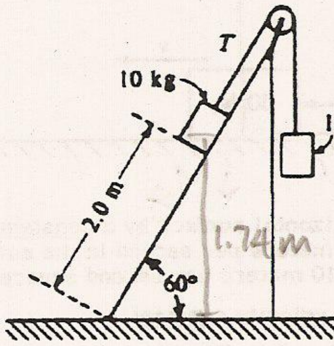
$KE_f = 180\text{J} + 60\text{J} = 240\text{J}$

$V_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{2 \cdot 240}{10}} = 6.93\ \text{m/s}$

ΔS 4m
V_0 6 m/s
V_f 6.93 m/s
a
t

SOLVE for a: $\frac{V_f^2 - V_0^2}{2s} = a = \frac{6.93^2 - 6^2}{8} = 1.5\ \text{m/s}^2$!

Problem 7



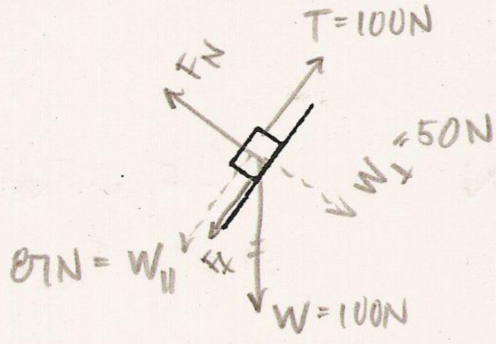
STATIC
$T = Mg$

2. Two 10-kilogram boxes are connected by a massless string that passes over a massless frictionless pulley as shown above. The boxes remain at rest, with the one on the right hanging vertically and the one on the left 2.0 meters from the bottom of an inclined plane that makes an angle of 60° with the horizontal. The coefficients of kinetic friction and static friction between the left-hand box and the plane are 0.15 and 0.30, respectively. You may use $g = 10\ m/s^2$, $\sin 60° = 0.87$, and $\cos 60° = 0.50$.

(a) What is the tension T in the string? $T = Mg = 100N$

(b) On the diagram below, draw and label all the forces acting on the box that is on the plane.

b)



c) $T > W_{\parallel}$ ∴ F_f DOWN INCLINE

$T = W_{\parallel} + F_f$

$F_f = T - W_{\parallel} = 100 - 87$

$= \boxed{13N}$

(c) Determine the magnitude of the frictional force acting on the box on the plane.

The string is then cut and the left-hand box slides down the inclined plane.

(d) Determine the amount of mechanical energy that is converted into thermal energy during the slide to the bottom.

(e) Determine the kinetic energy of the left-hand box when it reaches the bottom of the plane.

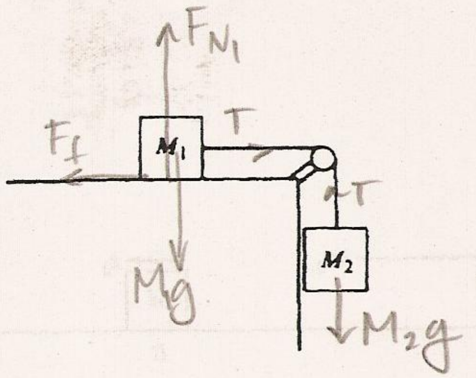
d) $W_{F_f} = F_f \cdot \Delta S_{\parallel} = \mu_k F_N \cdot \Delta S_{\parallel} = \mu_k W_{\perp} \Delta S_{\parallel} = 0.15(50)(2m)$

$= \boxed{15J}$

e) $KE_{BOT} = GPE_{TOP} - W_{F_f} = 10kg \cdot 10\frac{m}{s^2} \cdot 1.74m - 15J$

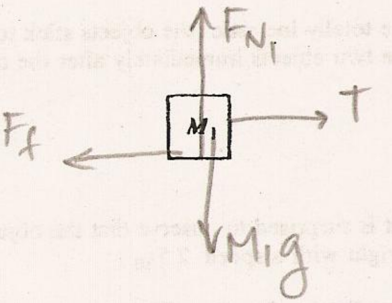
$= \boxed{159J}$

Problem 8



1. In the system shown above, the block of mass M_1 is on a rough horizontal table. The string that attaches it to the block of mass M_2 passes over a frictionless pulley of negligible mass. The coefficient of kinetic friction μ_k between M_1 and the table is less than the coefficient of static friction μ_s.

(a) On the diagram below, draw and identify all the forces acting on the block of mass M_1.



(b) In terms of M_1 and M_2 determine the minimum value of μ_s that will prevent the blocks from moving.

The blocks are set in motion by giving M_2 a momentary downward push. In terms of M_1, M_2, μ_k, and g, determine each of the following:

(c) The magnitude of the acceleration of M_1

(d) The tension in the string

b) $T = F_f = \mu_s M_1 g$ and $T = M_2 g$

$\therefore \; M_2 \cancel{g} = \mu_s M_1 \cancel{g}$

$$\boxed{\mu_s = \frac{M_2}{M_1}}$$

c) $M_2 g - T = M_2 a$

$T - F_f = T - \mu_k M_1 g = M_1 a$

$\therefore \; M_2 g - M_1 a - \mu_k M_1 g = M_2 a$

$$\boxed{a = \frac{M_2 g - \mu_k M_1 g}{M_2 + M_1}}$$

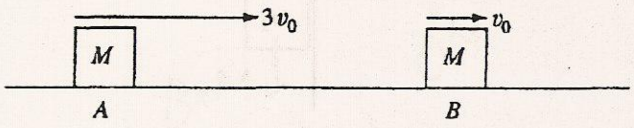
WHICH IS CONSISTENT W/ "SYSTEM APPROACH"

d) $T = M_1 a + \mu_k M_1 g$

$$T = M_1 \left(\frac{M_2 g - \mu_k M_1 g}{M_2 + M_1} \right) + \mu_k M_1 g$$

Problem 9

1996-B No. 1



1. (15 points)
Two identical objects A and B of mass M move on a one-dimensional, horizontal air track. Object B initially moves to the right with speed v_0. Object A initially moves to the right with speed $3v_0$, so that it collides with object B. Friction is negligible. Express your answers to the following in terms of M and v_0.

(a) Determine the total momentum of the system of the two objects.

(b) A student predicts that the collision will be totally inelastic (the objects stick together on collision). Assuming this is true, determine the following for the two objects immediately after the collision.

i. The speed

ii. The direction of motion (left or right)

When the experiment is performed, the student is surprised to observe that the objects separate after the collision and that object B subsequently moves to the right with a speed $2.5v_0$.

(c) Determine the following for object A immediately after the collision.

i. The speed
ii. The direction of motion (left or right)

(d) Determine the kinetic energy dissipated in the actual experiment.

a) $M(3v_0) + Mv_0 = \boxed{4Mv_0}$

b) i) $4Mv_0 = (M+M)v_f = 2Mv_f$ (ii) $\longrightarrow$ (v_f is $+$)

$\boxed{v_f = 2v_0}$

c) i) $4\cancel{M}v_0 = \cancel{M}v_A + \cancel{M}(2.5v_0)$ (ii) $\longrightarrow$ (v_A is $+$)

$\boxed{v_A = 1.5v_0}$

d) $\Delta KE = KE_f - KE_0$

$= \frac{1}{2}M(\frac{3}{2}v_0)^2 + \frac{1}{2}M(\frac{5}{2}v_0)^2 - \frac{1}{2}M(3v_0)^2 - \frac{1}{2}Mv_0^2$

$= \frac{9}{8}Mv_0^2 + \frac{25}{8}Mv_0^2 - 5Mv_0^2$

$= \frac{34}{8}Mv_0^2 - \frac{40}{8}Mv_0^2 = \boxed{-\frac{6}{8}Mv_0^2}$

Problem 10

1992-B No.2

2. A 30-kilogram child moving at 4.0 meters per second jumps onto a 50-kilogram sled that is initially at rest on a long, frictionless, horizontal sheet of ice.

(a) Determine the speed of the child-sled system after the child jumps onto the sled.

(b) Determine the kinetic energy of the child-sled system after the child jumps onto the sled.

After coasting at constant speed for a short time, the child jumps off the sled in such a way that she is at rest with respect to the ice.

(c) Determine the speed of the sled after the child jumps off it.

(d) Determine the kinetic energy of the child-sled system when the child is at rest on the ice.

(e) Compare the kinetic energies that were determined in parts (b) and (d). If the energy is greater in (d) than it is in (b), where did the increase come from? If the energy is less in (d) than it is in (b), where did the energy go?

a) CONS OF $\vec{p}$

$\vec{p}_o = \vec{p}_f$

$30\,kg \cdot 4\frac{m}{s} = 80\,kg \cdot v_f$; $\boxed{v_f = 1.5\frac{m}{s}}$

b) $KE = \frac{1}{2}mv^2 = \frac{1}{2}(80\,kg)(1.5)^2 = \boxed{90\,J}$

c) CONS OF $\vec{p}$

$\vec{p}_o = \vec{p}_f$

$120\,kg \cdot \frac{m}{s} = 50\,kg \cdot v_f + \cancel{30\,kg \cdot 0\frac{m}{s}}^{\,0}$

$\boxed{v_f = 2.4\,m/s}$

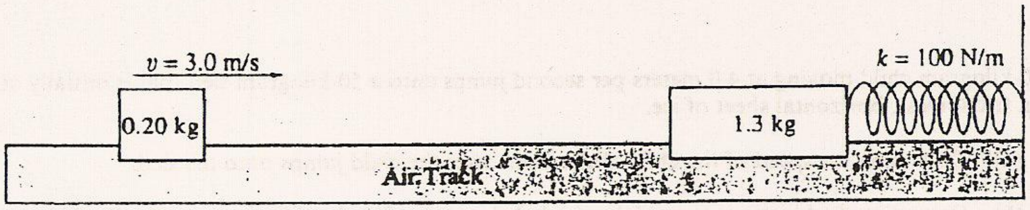
d) $\frac{1}{2}(50\,kg)(2.4)^2 = \boxed{144\,J}$

SLED ONLY

e) KE↑ (ΔKE +) AS WORK IS DONE ON SLED BY CHILD (CHEM ENERGY → MECH ENERGY)

Problem 11

1995-B No. 1



1. (15 points)
As shown above, a 0.20-kilogram mass is sliding on a horizontal, frictionless air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a massless spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

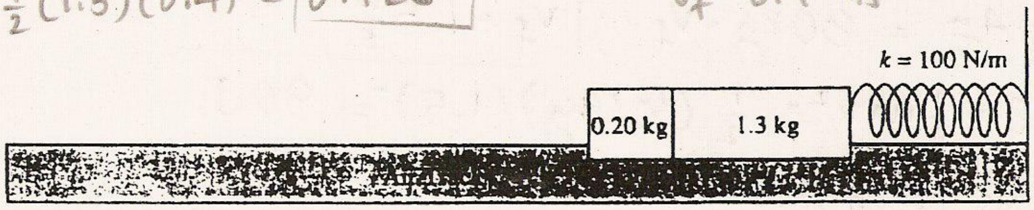
(a) Determine the following for the 0.20-kilogram mass immediately before the impact.
  i. Its linear momentum $0.20\,kg \cdot 3\,m/s = \boxed{0.6\ kg\text{-}m/s}$
  ii. Its kinetic energy $\frac{1}{2}(0.2)(3\,m/s)^2 = \boxed{0.9\,J}$

(b) Determine the following for the combined masses immediately after the impact.
  i. The linear momentum SAME AS ABOVE (0.6 kgm/s) — MOMENTUM OF SYSTEM CONSERVED
  ii. The kinetic energy

$P_f = 1.5\,kg\ V_f = 0.6$

$V_f = 0.4\,m/s$

$\frac{1}{2}(1.5)(0.4)^2 = \boxed{0.12\,J}$



After the collision, the two masses undergo simple harmonic motion about their position at impact.

(c) Determine the amplitude of the harmonic motion.

(d) Determine the period of the harmonic motion.

$T = 2\pi\sqrt{\frac{m}{K}}$

$= 2\pi\sqrt{\frac{1.5}{100}} = \boxed{0.77\ SEC}$

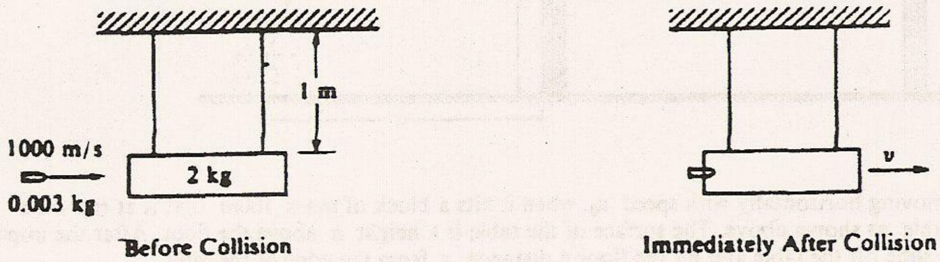
c) @ S_{MAX}, $V = 0$ all KE → EPE (0.12J)

$\frac{1}{2}KS_{MAX}^2 = 0.12J$

$S_{MAX} = \sqrt{\frac{2 \cdot 0.12}{100}} = \boxed{0.049\,m}$

Problem 12

1985-B NO.1



1. A 2-kilogram block initially hangs at rest at the end of two 1-meter strings of negligible mass as shown on the left diagram above. A 0.003-kilogram bullet, moving horizontally with a speed of 1000 meters per second, strikes the block and becomes embedded in it. After the collision, the bullet/block combination swings upward, but does not rotate.

   (a) Calculate the speed v of the bullet/block combination just after the collision.

   (b) Calculate the ratio of the initial kinetic energy of the bullet to the kinetic energy of the bullet/block combination immediately after the collision.

   (c) Calculate the maximum vertical height above the initial rest position reached by the bullet/block combination.

a) $m v_B + \cancel{M v_{BL}}^{\approx 0} = (m + M) V_{B+B}$

$0.003\text{ kg} \cdot 1000 \frac{m}{s} = 2.003\text{ kg}\ V_{B+B}$

$\boxed{V_{B+B} = \frac{3.00}{2.003} \approx 1.5 \frac{m}{s}}$

b) $KE_0 = \frac{1}{2}(0.003)(1000)^2 = 1500\text{J}$

$KE_f = \frac{1}{2}(2.003)(1.5)^2 = 2.253\text{J}$

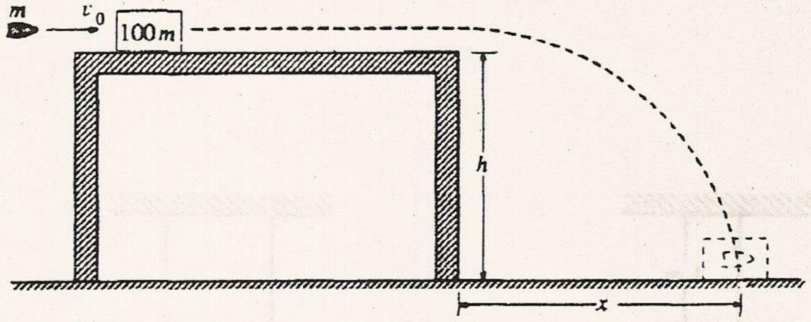
$= \boxed{\frac{666}{1}}$

c) $KE_f \longrightarrow GPE$

$mgh = 2.253\text{J}$

$\boxed{h = \frac{2.253}{2.003 \cdot 10} = 0.112\text{ m}}$

Problem 13



1. A bullet of mass m is moving horizontally with speed v_0 when it hits a block of mass $100m$ that is at rest on a horizontal frictionless table, as shown above. The surface of the table is a height h above the floor. After the impact the bullet and the block slide off the table and hit the floor a distance x from the edge of the table.

Derive expressions for the following quantities in terms of m, h, v_0, and appropriate constants:

(a) the speed of the block as it leaves the table

(b) the change in kinetic energy of the bullet-block system during impact

(c) the distance x

Suppose that the bullet passes through the block instead of remaining in it.

(d) State whether the time required for the block to reach the floor from the edge of the table would now be greater, less, or the same. Justify your answer.

(e) State whether the distance x for the block would now be greater, less, or the same. Justify your answer.

a) $mv_0 = (100m + m)v_f = 101 m v_f$; $\boxed{v_f = \frac{v_0}{101}}$

b) $\Delta KE = KE_f - KE_0$

$= \frac{1}{2}(101m)\left(\frac{v_0}{101}\right)^2 - \frac{1}{2}mv_0^2$

$= \frac{1}{2}\frac{mv_0^2}{101} - \frac{1}{2}mv_0^2 = \boxed{\frac{-100}{202}mv_0^2}$

c)

	x	y
Δs	$\boxed{}$	$-h$
v_0	$\frac{v_0}{101}$	0
v_f	$\frac{v_0}{101}$	$\boxed{-\sqrt{2gh}}$
a	×	$-g$
t	$\boxed{}$	$= \boxed{}$

$v_f^2 = \cancel{v_0^2}^{\,0} + 2a\Delta s = 2gh$, $\bar{v} = \frac{-\sqrt{2gh}}{2}$

$\Delta s = \bar{v}\Delta t$; $\Delta t = \frac{\Delta s}{\bar{v}} = \frac{-h}{\left(\frac{-\sqrt{2gh}}{2}\right)} = \frac{2h\sqrt{2gh}}{2gh} = \boxed{\frac{\sqrt{2gh}}{g}} = t$

$\therefore \Delta s_x = \bar{v}\Delta t$

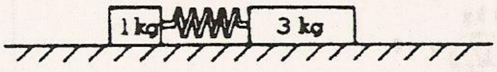
$= \frac{v_0}{101} \cdot \frac{\sqrt{2gh}}{2g} = \boxed{\frac{v_0\sqrt{2gh}}{202g}}$

d) HORIZ. SPEED DOES NOT AFFECT RATE OF VERTICAL ACCEL ⟶ NO CHANGE IN TIME

e) BY PASSING THROUGH BLOCK HAS LESS OF p TO RIGHT. SLOWER BLOCK WILL TRAVEL LESS DISTANCE HORIZONTALLY

Problem 14

1981-B NO.2



2. A massless spring is between a 1-kilogram mass and a 3-kilogram mass as shown above, but is not attached to either mass. Both masses are on a horizontal frictionless table. In an experiment, the 1-kilogram mass is held in place, and the spring is compressed by pushing on the 3-kilogram mass. The 3-kilogram mass is then released and moves off with a speed of 10 meters per second.

(a) Determine the minimum work needed to compress the spring in this experiment.

The spring is compressed again exactly as above, but this time both masses are released simultaneously.

(b) Determine the final velocity of each mass relative to the table after the masses are released.

a) $W_{\text{TO COMPRESS}} = EPE \longrightarrow KE$

$\therefore KE = \frac{1}{2}mv^2 = \frac{1}{2}(3)(10)^2 = \boxed{150J = EPE_0}$

b) $\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_3 v_3^2 = 150J$ } CONS. OF ENERGY

$m_1 v_1 - m_3 v_3 = 0$ } CONS. OF $\vec{p}$

$v_1 - 3v_3 = 0$

SUB-IN $v_1 = 3v_3$

$\frac{1}{2}m_1(3v_3)^2 + \frac{1}{2}m_3 v_3^2 = 150$

$\frac{9v_3^2}{2} + \frac{3v_3^2}{2} = 150J$

$6v_3^2 = 150J$

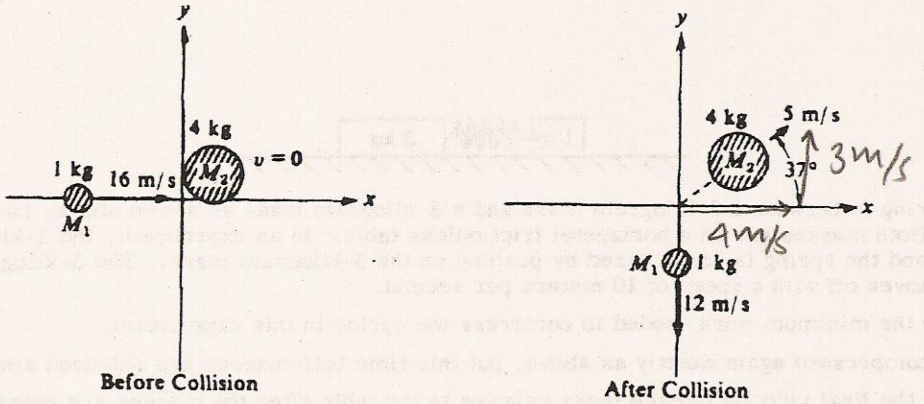
$v_3^2 = 25$

$v_3 = 5\frac{m}{s} \rightarrow$

and

$v_1 = 15\frac{m}{s} \leftarrow$

opposite directions



View From Above

2. Two objects of masses M_1 = 1 kilogram and M_2 = 4 kilograms are free to slide on a horizontal frictionless surface. The objects collide and the magnitudes and directions of the velocities of the two objects before and after the collision are shown on the diagram above. ($\sin 37° = 0.6$, $\cos 37° = 0.8$, $\tan 37° = 0.75$)

(a) Calculate the x and y components (p_x and p_y, respectively) of the momenta of the two objects before and after the collision, and write your results in the proper places in the following table.

	M_1 = 1 kg		M_2 = 4 kg	
	$p_x\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)$	$p_y\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)$	$p_x\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)$	$p_y\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)$
Before Collision	16	0	0	0
After Collision	0	-12	$4 \cdot 4\frac{m}{s} = +16$	$4 \cdot 3\frac{m}{s} = +12$

b) $\Sigma\vec{P}_{x0} = \Sigma\vec{P}_{xf}$ and $\Sigma\vec{P}_{y0} = \Sigma\vec{P}_{yf}$

(b) Show, using the data that you listed in the table, that linear momentum is conserved in this collision.

(c) Calculate the kinetic energy of the two-object system before and after the collision.

(d) Is kinetic energy conserved in the collision?

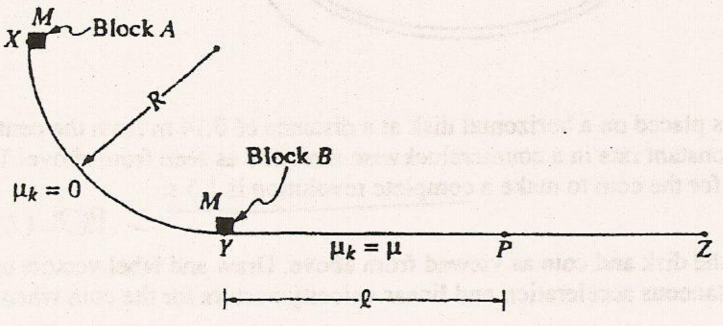
c) $KE_0 = \frac{1}{2}(1)(16)^2 = \boxed{128 J}$

$KE_f = \frac{1}{2}(1)(12)^2 + \frac{1}{2}(4)(5)^2 = 72 + 50 = \boxed{122 J}$

d) $\longrightarrow$ $KE_f < KE_0$ KE NOT CONSERVED

Problem 16

1994-B No. 2



Side View

2. A track consists of a frictionless arc XY, which is a quarter-circle of radius R, and a rough horizontal section YZ. Block A of mass M is released from rest at point X, slides down the curved section of the track, and collides instantaneously and inelastically with identical block B at point Y. The two blocks move together to the right, sliding past point P, which is a distance ℓ from point Y. The coefficient of kinetic friction between the blocks and the horizontal part of the track is μ. Express your answers in terms of M, ℓ, μ, R, and g.

(a) Determine the speed of block A just before it hits block B.

(b) Determine the speed of the combined blocks immediately after the collision.

(c) Determine the amount of kinetic energy lost due to the collision.

(d) The specific heat of the material used to make the blocks is c. Determine the temperature rise that results from the collision in terms of c and the other given quantities. (Assume that no energy is transferred to the track or to the air surrounding the blocks.)

(e) Determine the additional thermal energy that is generated as the blocks move from Y to P.

a) CONS. OF ENERGY $\cancel{M}gR \rightarrow \frac{1}{2}\cancel{M}v_A^2$ $\boxed{v_A = \sqrt{2gR}}$

b) $\vec{P}_0 = \vec{P}_f \rightarrow \cancel{M}\sqrt{2gR} = 2\cancel{M}v_{A+B}$; $\boxed{v_{A+B} = \frac{\sqrt{2gR}}{2}}$

c) $\Delta KE = KE_f - KE_0 = \frac{1}{2}2M\left(\frac{\sqrt{2gR}}{2}\right)^2 - \frac{1}{2}M\left(\sqrt{2gR}\right)^2$

$= \frac{2gR}{4} - \frac{1}{2}(2gR) = \boxed{-\frac{1}{2}MgR}$

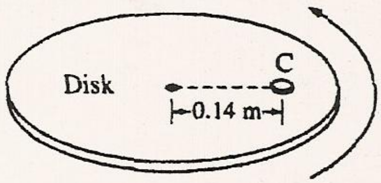
e) FIND $W_{F_f} = -F_f \cdot \Delta S_{\parallel}$

$\boxed{= -\mu 2Mg \cdot \ell}$

NEG. SIGN FOR LOSS BY MECH. SYSTEM.

Problem 17

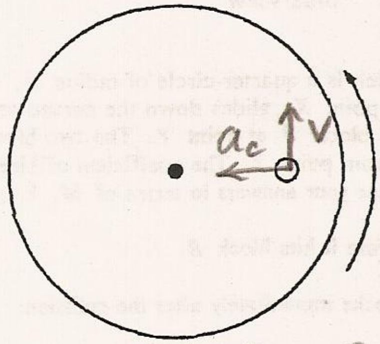
1999-B No. 5



5. (10 points)
A coin C of mass 0.0050 kg is placed on a horizontal disk at a distance of 0.14 m from the center, as shown above. The disk rotates at a constant rate in a counterclockwise direction as seen from above. The coin does not slip, and the time it takes for the coin to make a complete revolution is 1.5 s.

PERIOD

(a) The figure below shows the disk and coin as viewed from above. Draw and label vectors on the figure below to show the instantaneous acceleration and linear velocity vectors for the coin when it is at the position shown.

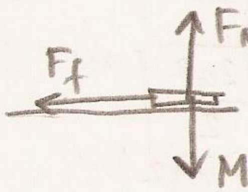


(b) Determine the linear speed of the coin. $v = \frac{2\pi R}{T} = \frac{2\pi(0.14\,m)}{1.5\,sec} = \boxed{0.59\,m/s}$

(c) The rate of rotation of the disk is gradually increased. The coefficient of static friction between the coin and the disk is 0.50. Determine the linear speed of the coin when it just begins to slip.

(d) If the experiment in part (c) were repeated with a second, identical coin glued to the top of the first coin, how would this affect the answer to part (c) ? Explain your reasoning.

c)



$F_f = \mu F_N = \mu M g \qquad F_f = \frac{Mv^2}{R}$

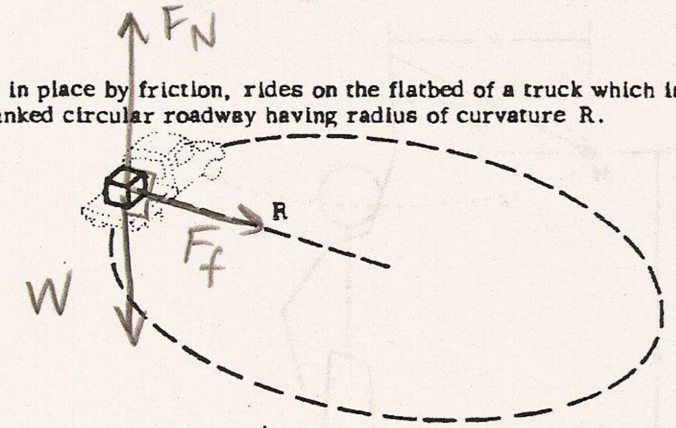
$\mu M g = \frac{Mv^2}{R}\ ; \ v = \sqrt{\mu g R}$

$= \sqrt{0.5 \cdot 10 \cdot 0.14} = \boxed{0.84\,m/s}$

d) NOTE $v = \sqrt{\mu g R}$ HAS NO MASS DEPENDENCE $\therefore$ DOUBLING MASS HAS NO EFFECT ON VALUE IN (c)

Problem 18

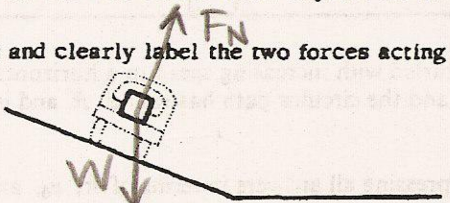
2. A box of mass M, held in place by friction, rides on the flatbed of a truck which is traveling with constant speed v. The truck is on an unbanked circular roadway having radius of curvature R.



(a) On the diagram provided above, indicate and clearly label all the force vectors acting on the box.
(b) Find what condition must be satisfied by the coefficient of static friction μ between the box and the truck bed. Express your answer in terms of v, R, and g.

If the roadway is properly banked, the box will still remain in place on the truck for the same speed v even when the truck bed is frictionless.

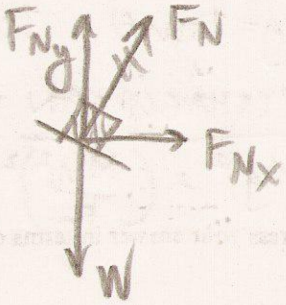
(c) On the diagram below, indicate and clearly label the two forces acting on the box under these conditions.



(d) Which, if either, of the two forces acting on the box is greater in magnitude? Explain.

b) $F_f = \mu \cancel{M} g = \frac{\cancel{M} v^2}{R} \longrightarrow \boxed{\mu = \frac{v^2}{gR}}$

d)



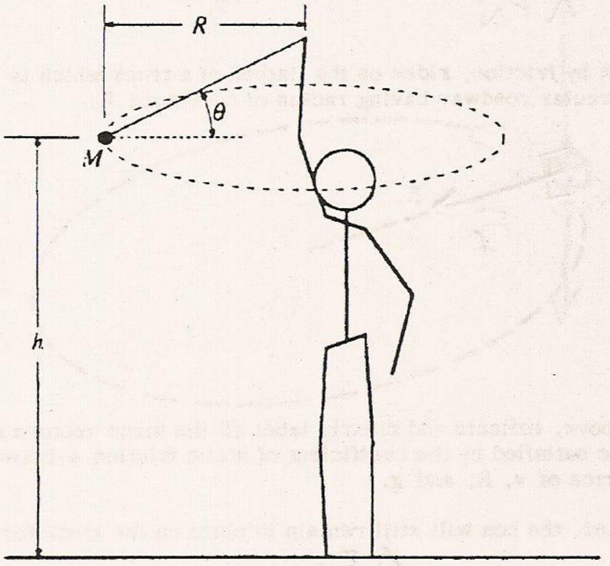
$F_{N_y} = W$ } NEWTON'S 1st

$F_{N_y} < F_N$ } component less than total vector

$\therefore W < F_N$

Problem 19

1989–B NO. 1

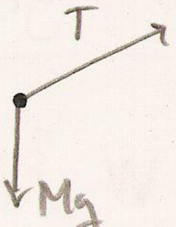


An object of mass M on a string is whirled with increasing speed in a horizontal circle, as shown above. When the string breaks, the object has speed v_0 and the circular path has radius R and is a height h above the ground. Neglect air friction.

(a) Determine the following, expressing all answers in terms of h, v_0, and g.

i. The time required for the object to hit the ground after the string breaks
ii. The horizontal distance the object travels from the time the string breaks until it hits the ground
iii. The speed of the object just before it hits the ground

(b) On the figure below, draw and label all the forces acting on the object when it is in the position shown in the diagram above.



c) $T_x = T\sin\theta$, $T_y = T\cos\theta$, Mg

$Mg = T\sin\theta \quad T\cos\theta = \frac{Mv_0^2}{R}$

BY PYTHAGOREAN THEOREM

$T = \sqrt{(Mg)^2 + \left(\frac{Mv_0^2}{R}\right)^2}$

(c) Determine the tension in the string just before the string breaks. Express your answer in terms of M, R, v_0, and g.

a) 2D projectile

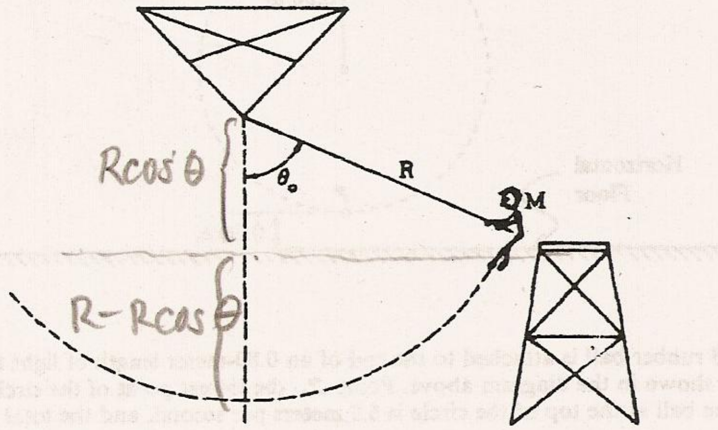
	x	y
ΔS	$\boxed{}$	$-h$
v_0	v_0	0
v_f	v_0	$\boxed{\sqrt{2gh}} \leftarrow v_f^2 = \cancel{v_0^2}^{\,0} + 2(-g)(-h)$
a	0	$-g$
t	$\boxed{}$	$= \boxed{} \leftarrow \boxed{t = \frac{v_f - \cancel{v_0}^{\,0}}{a} = \frac{\sqrt{2gh}}{g}}$

$\boxed{\Delta S_x = V_x \Delta t = \frac{V_0\sqrt{2gh}}{h}}$

SPEED = v_0, $\sqrt{2gh}$, v

$\boxed{V = \sqrt{V_0^2 + 2gh}}$

Problem 20



3. A child of mass M holds onto a rope and steps off a platform. Assume that the initial speed of the child is zero. The rope has length R and negligible mass. The initial angle of the rope with the vertical is θ_0, as shown in the drawing above.

(a) Using the principle of conservation of energy, develop an expression for the speed of the child at the lowest point in the swing in terms of g, R, and cos θ_0.

(b) The tension in the rope at the lowest point is 1.5 times the weight of the child. Determine the value of cos θ_0.

a) $\cancel{M}g(R - R\cos\theta) = \frac{1}{2}\cancel{M}v_{BOT}^2$

$$\boxed{v_{BOT} = \sqrt{2g(R - R\cos\theta)}}$$

b) @ BOT

T, Mg

$T = 1.5Mg$

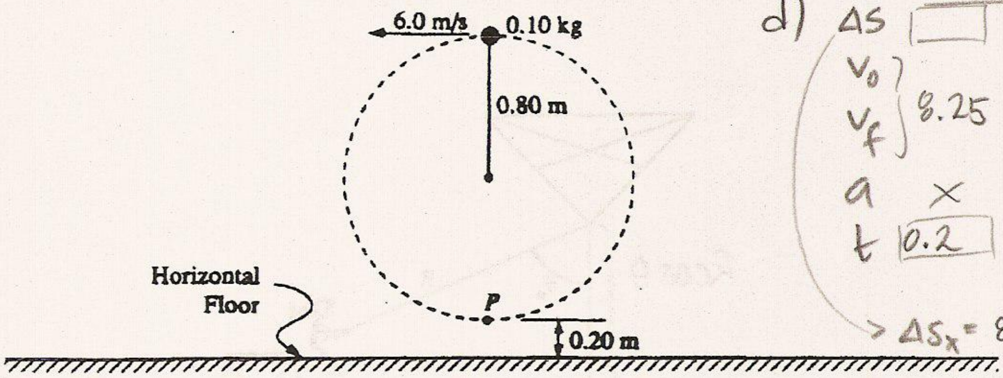
$$T - Mg = \frac{Mv_{BOT}^2}{R} = \frac{3}{2}Mg - Mg = \frac{1}{2}Mg$$

$$\frac{1}{2}\cancel{M}\cancel{g} = \frac{\cancel{M}(2\cancel{g})(R - R\cos\theta)}{R} \longrightarrow \frac{1}{2} = \frac{2\cancel{R}(1 - \cos\theta)}{\cancel{R}}$$

$$\frac{1}{4} = 1 - \cos\theta \quad \therefore \boxed{\cos\theta = \frac{3}{4}}$$

$\theta = \cos^{-1}\left(\frac{3}{4}\right) = 41.4°$

Problem 21



d)

	x	y
Δs	$\boxed{}$	$-0.20m$
v_0	8.25	0
v_f		$\boxed{-2}$
a	$\times$	-10
t	$\boxed{0.2}$	$= \boxed{0.2}$

$v_f^2 = \cancel{v_0^2}^{\,0} + 2a\Delta s$
$= 2(-10)(-0.2)$
$v_f = -2\ m/s$

$\Delta s_x = 8.25\frac{m}{s} \cdot 0.2\,sec$
$= \boxed{1.65m}$

1. A 0.10-kilogram solid rubber ball is attached to the end of an 0.80-meter length of light thread. The ball is swung in a vertical circle, as shown in the diagram above. Point P, the lowest point of the circle, is 0.20 meter above the floor. The speed of the ball at the top of the circle is 6.0 meters per second, and the total energy of the ball is kept constant.

(a) Determine the total energy of the ball, using the floor as the zero point for gravitational potential energy.

(b) Determine the speed of the ball at point P, the lowest point of the circle.

(c) Determine the tension in the thread at
   i. the top of the circle;
   ii. the bottom of the circle.

The ball only reaches the top of the circle once before the thread breaks when the ball is at the lowest point of the circle.

(d) Determine the horizontal distance that the ball travels before hitting the floor.

a) $TE = KE + GPE = \frac{1}{2}(0.1)(6\frac{m}{s})^2 + 0.1(10)(2(0.8)+0.2)$
$= 1.8J + 1.8J = \boxed{3.6J}$

b) $\Delta GPE = \Delta KE$
$\rightarrow 0.1(10)(-1.6) = -1.6J$ $\therefore$ $KE = 1.8 + 1.6 = 3.4J$
DOWN
$\frac{1}{2}mv_{BOT}^2 = KE$
$v_{BOT} = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \cdot 3.4J}{0.1}} = \boxed{8.25\ m/s}$

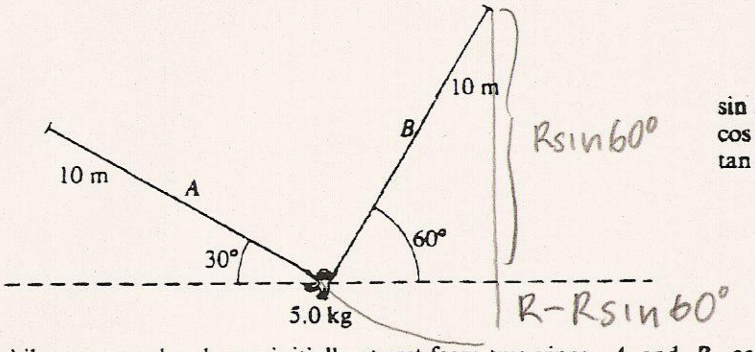
c) i) Mg, T

$\frac{mv_{TOP}^2}{R} = Mg + T_{TOP}$; $T_{TOP} = \frac{Mv_{TOP}^2}{R} - Mg = \frac{0.1(6^2)}{0.8} - 1 = \boxed{3.5N}$

ii) $T_{BOT} - Mg = \frac{Mv_{BOT}^2}{R}$; $T_{BOT} = \frac{Mv_B^2}{R} + Mg = \frac{0.1(8.25)^2}{0.8} + 1 = \boxed{9.5N}$

Problem 22

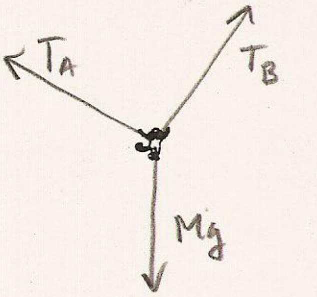
1991-B NO.1



sin 30° = 0.50	sin 60° = 0.87
cos 30° = 0.87	cos 60° = 0.50
tan 30° = 0.58	tan 60° = 1.73

A 5.0-kilogram monkey hangs initially at rest from two vines, A and B, as shown above. Each of the vines has length 10 meters and negligible mass.

(a) On the figure below, draw and label all of the forces acting on the monkey. (Do not resolve the forces into components, but do indicate their directions.)



d) $T - Mg = \frac{Mv_{BOT}^2}{R}$

$T = \frac{Mv_B^2}{R} + Mg$

$= \frac{5(\sqrt{26})^2}{10} + 50$

$\boxed{= 63\text{ N}}$

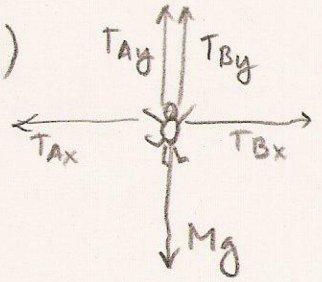
(b) Determine the tension in vine B while the monkey is at rest.

The monkey releases vine A and swings on vine B. Neglect air resistance.

(c) Determine the speed of the monkey as it passes through the lowest point of its first swing.

(d) Determine the tension in vine B as the monkey passes through the lowest point of its first swing.

b)



$T_{Bx} = T_{Ax}$ $\quad T_{Ay} + T_{By} = Mg = 50N$

$T_B\cos 60 = T_A\cos 30°$ $\quad T_A\sin 30° + T_B\sin 60° = 50$

$0.50T_B = 0.87T_A$ $\quad 0.5T_A + 0.87T_B = 50$

$T_A = \frac{0.50}{0.87}T_B$ $\quad T_B\frac{0.5(0.5)}{0.87} + 0.87T_B = 50$

SUB IN

$1.157T_B = 50$

$\boxed{T_B = 43.5N}$

c) $GPE_0 \rightarrow KE_f$

$\cancel{M}gh = \frac{1}{2}\cancel{M}v^2$

$V_{BOT} = \sqrt{2gh}$

$= \sqrt{2g(1.3)} = \sqrt{26} \sim \boxed{5 \text{ m/s } (5.1 \text{ m/s})}$

$h = R - R\sin 60$

$= 10 - 10(0.87)$

$= 1.3m$