Calculus Review Worksheet

(1) Find $\frac{dy}{dx}$ and simplify please.

(a) $y = 5x^3 - \frac{7}{x} + \frac{2}{x^2}$

(b) $y = \left(3x^2 - 4\right)^3$

(c) $y = (4x+7)^3(9x-2)^2$

(d) $y = \frac{x^2 - 1}{x^2 + 1}$

(e) $y = \frac{\left(x^2 - 2\right)^2}{x-1}$

(f) $y = \frac{4}{\sqrt{x^2 - 2x}}$

(g) $3x^2y^3 - 2x^3y^2 + 5x - y = 7$

(h) $\sqrt[3]{x^2} + \sqrt[3]{y^2} = 1$

(i) $y = 2\sin\left(\sqrt{1 - x}\right)$

(j) $y = 3\cos^4\left(5x^2\right)$

(k) $y = x^2 \tan\left(x^2\right)$

(l) $y = \csc^2(5x) - \cot^2(5x)$

(m) $y = \frac{1 + 2\sin(x)}{1 - 2\sin(x)}$

(n) $x = 1 - \cos(3t)$
$y = 1 + \sin(2t)$

(o) $y = \sin(3x)\cos(2x)$

(p) $y = \sin^3\left(\cos^2\left(\sqrt{x}\right)\right)$

(2) Find the coordinates of all points on the graph of each of the following where the tangents to the graph are horizontal.

(a) $y = x^3 + 3x^2 - 9x - 27$

(b) $y = \tan(x) - 2x + 1$ if $0 \le x < 2\pi$.

(3) Find $\frac{dy}{dx}$ using the definition of the derivative given the function $f(x) = \frac{4}{\sqrt{2x + 3}}$

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(4) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ please. $\sqrt{x} + \sqrt{y} = 1$

(5) Find the values of the constants **a** and **b** so that the graphs of the curves $y = 3x - x^2$ and $y = x^2 + \mathbf{a}x + \mathbf{b}$ shall be tangent to each other at the point (3 , 0).

(6) Find the coordinates of all points on the graph of $x^2 - xy + 2y^2 = 28$ where the tangents to the graph are; **(a)** horizontal, and **(b)** vertical. Use a graphing calculator to sketch a graph of the curve.

(7) Write the equation of the line tangent to each of the following at the indicated point.

(a) $y = 2x^3 - 3x^2 + 4x$ at $x = 1$.

(b) $x^2 - 2xy + y^2 - 2x + y = 10$ at $(2 , -2)$.

(c) $y = 2\sin(3x) - \cos(2x)$ at $x = \pi$.

(d) $x = \sqrt[3]{6t + 2}$, $y = 3t^2 - 2t + 5$ at $t = 1$.

(8) Find $\frac{dy}{dx}$ if $y = f^2\left(\frac{1}{x}\right)$ and $f'(x) = \sec(3x)$.

(9) Find the value of $\frac{dy}{dx}$ at $x = 1$ for the function $y = f^3\left(g^2(x)\right)$ if $g(1) = 2$, $g'(1) = 3$, $f(1) = 6$, $f'(1) = 7$, $f(4) = -1$, $f'(4) = 5$.

(10) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t please. Find all points on the graph where the tangents are vertical.

$$x = 3t^2 - 2t - 3 \quad , \quad y = t^2 + 2t - 4 .$$

(11) Evaluate the following limit please. $\lim_{h \to 0}\left(\frac{\cot(4x + 4h) - \cot(4x)}{h} \right)$

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Answers

(1) **(a)** $\frac{dy}{dx} = 15x^2 + 7x^{-2} - 4x^{-3}$

(b) $\frac{dy}{dx} = 18x\left(3x^2 - 4\right)^2$

(c) $\frac{dy}{dx} = 6(4x+7)^2(9x-2)(30x+17)$

(d) $\frac{dy}{dx} = \frac{4x}{\left(x^2+1\right)^2}$

(e) $\frac{dy}{dx} = \frac{\left(x^2-2\right)\left(3x^2-4x+2\right)}{(x-1)^2}$

(f) $\frac{dy}{dx} = \frac{-4(x-1)}{\left(x^2-2x\right)^{\frac{3}{2}}}$

(g) $\frac{dy}{dx} = \frac{-6xy^3 + 6x^2y^2 - 5}{9x^2y^2 - 4x^3y - 1}$

(h) $\frac{dy}{dx} = \frac{-y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$

(i) $\frac{dy}{dx} = -\frac{\cos\left(\sqrt{1-x}\right)}{\sqrt{1-x}}$

(j) $\frac{dy}{dx} = 2x\tan(x^2) + 2x^3\sec^2(x^2)$

(k) $\frac{dy}{dx} = 2x\tan\left(x^2\right) + 2x^3\sec^2\left(x^2\right)$

(l) $\frac{dy}{dx} = 0$

(m) $\frac{dy}{dx} = \frac{4\cos(x)}{(1-2\sin(x))^2}$

(n) $\frac{dy}{dx} = \frac{2\cos(2t)}{3\sin(3t)}$

(o) $\frac{dy}{dx} = 3\cos(3x)\cos(2x) - 2\sin(2x)\sin(3x)$

(p) $\frac{dy}{dx} = \frac{-3\sin^2\left(\cos^2\left(\sqrt{x}\right)\right)\cos\left(\cos^2\left(\sqrt{x}\right)\right)\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)}{\sqrt{x}}$

(2) **(a)** $(-3, 0)$, $(1, -32)$

(b) $\left(\frac{\pi}{4}, 2-\frac{\pi}{2}\right)$, $\left(\frac{3\pi}{4}, -\frac{3\pi}{2}\right)$, $\left(\frac{5\pi}{4}, 2-\frac{5\pi}{2}\right)$, $\left(\frac{7\pi}{4}, -\frac{7\pi}{2}\right)$

(3) $\frac{dy}{dx} = \frac{-4}{(2x+3)\sqrt{2x+3}}$

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Answers

(4) $\dfrac{dy}{dx} = \dfrac{-y^{\frac{1}{2}}}{x^{\frac{1}{2}}}$ $\qquad\qquad$ $\dfrac{d^2y}{dx^2} = \dfrac{1}{2x^{\frac{3}{2}}}$

(5) $\mathbf{a} = -9$, $\mathbf{b} = 18$

(6) **(a)** horizontal tangents at $(2\ ,\ 4)$ and $(-2\ ,\ -4)$

(b) vertical tangents at $(4\sqrt{2}\ ,\ \sqrt{2})$ and $(-4\sqrt{2}\ ,\ -\sqrt{2})$

(7) **(a)** $y = 4x - 1$

(b) $y + 2 = \frac{6}{7}(x - 2)$ or $y = \frac{6}{7}x - \frac{26}{7}$

(c) $y + 1 = -6(x - \pi)$

(d) $y = 8x - 10$

(8) $\dfrac{dy}{dx} = -\dfrac{2}{x^2} f\left(\dfrac{1}{x}\right) \sec\left(\dfrac{3}{x}\right)$

(9) 180

(10) $\dfrac{dy}{dx} = \dfrac{t + 1}{3t - 1}$, $\dfrac{d^2y}{dx^2} = \dfrac{-2}{(3t - 1)^3}$, vertical tangent at $\left(-\frac{10}{3}\ ,\ -\frac{29}{9}\right)$

(11) $-4\csc^2(4x)$