Calculus Assignment # 3

(1) Solve each of the following differential equations please.

(a) $\dfrac{dy}{dx} = \sqrt{xy}$, $x > 0$, $y > 0$

(b) $\dfrac{dy}{dx} = \dfrac{\sqrt{x + 1}}{y\sqrt{y^2 + 2}}$

(c) $\dfrac{dy}{dx} = \sqrt[3]{xy + 2x + 3y + 6}$

(d) $\dfrac{dy}{dt} = \left(2t + t^{-1}\right)^2$

(e) $\dfrac{dy}{dx} = \dfrac{1}{x^2 + 2x + 1}$

(f) $\dfrac{dx}{dt} = \sqrt{\left(t^2 - 2t^{-2}\right)^2 + 8}$

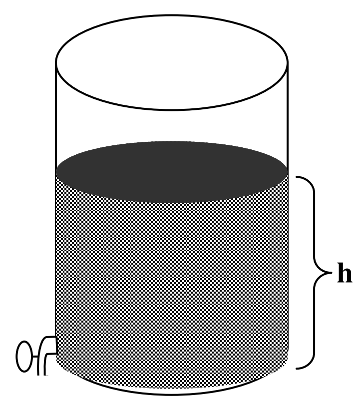
(2) Solve each of the following differential equations. Use the given information to evaluate the constant(s) of integration please.

(a) $\dfrac{dy}{dx} = 2xy^2$, $y = 1$ when $x = 1$.

(b) $\dfrac{dy}{dx} = \dfrac{4}{y}\left(\sqrt{\left(1 + y^2\right)^3}\right)$, $y = 0$ when $x = 0$

(c) $\dfrac{d^2y}{dx^2} = \dfrac{3x}{8}$, the graph of y passes through (4 , 4) with slope 3.

(3) If we open the drain in the cylindrical tank shown to the right, the water will flow out quickly when the tank is full, but will slow down as the tank drains. The rate at which the water level drops is proportional to the square root of the water's depth; $\dfrac{dh}{dt} = -k\sqrt{h}$. The value of k depends on the acceleration of gravity, the cross-section of the tank, and the cross-section area of the drain hole.



(a) Solve the equation for h as a function of t.

(b) Suppose t is measured in minutes and $k = \frac{1}{10}$. Find h as a function of t if $h = 9$ feet when $t = 0$.

(c) How long does it take the tank to drain if the water is 9 ft. deep to start with?

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Answers

(1) **(a)** $y^{\frac{1}{2}} = \frac{1}{3}x^{\frac{3}{2}} + C$

(b) $\left(y^2 + 2\right)^{\frac{3}{2}} = 2\left(x + 1\right)^{\frac{3}{2}} + C$

(c) $2\left(y + 2\right)^{\frac{2}{3}} = \left(x + 3\right)^{\frac{4}{3}} + C$

(d) $y = \frac{4}{3}t^3 + 4t - t^{-1} + C$

(e) $y = \dfrac{-1}{x + 1} + C$

(f) $x = \frac{1}{3}t^3 - 2t^{-1} + C$

(2) **(a)** $\frac{1}{y} = -x^2 + 2$

(b) $\dfrac{1}{\sqrt{1 + y^2}} = -4x + 1$

(c) $y = \frac{1}{16}x^3$

(3) **(a)** $h = \frac{1}{4}\left(C - kt\right)^2$

(b) $h = \frac{1}{4}\left(6 - \frac{1}{10}t\right)^2$

(c) 60 minutes