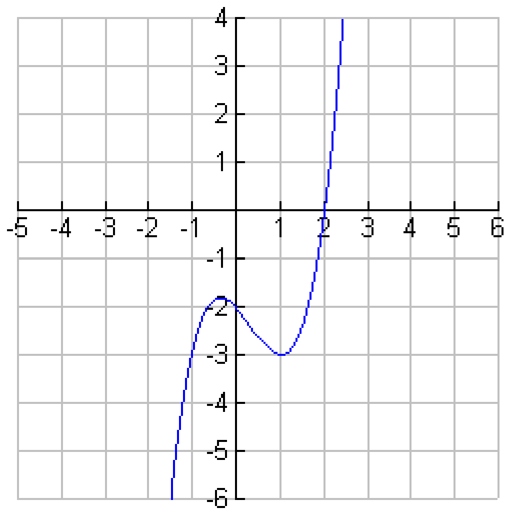
IVT, MVT and ROLLE'S THEOREM

IVT – Intermediate Value Theorem

What it says: If f is continuous on the closed interval [a, b] and k is a number between f(a) and f(b), then there is at least one number c in [a, b] such that f(c) = k

What it means: If f is continuous between two points, and f(a) = j and f(b) = k, then for any c between a and b, f(c) will take on a value between j and k.



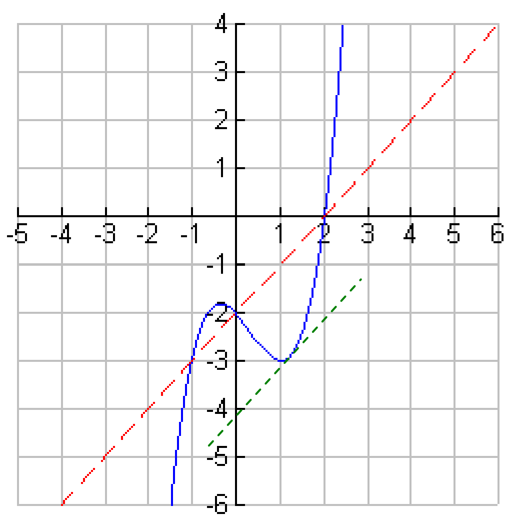
Notice in the picture: f(-1) = -3 and f(2) = 0. According to the IVT, somewhere between -1 and 2, there will be someplace where f(c) = -2 (or -1, or -½…)

When to use it: Use to prove that a particular intermediate y value when you know two other y values on a continuous function. NOT with derivatives!!

MVT – Mean Value Theorem

What it says: If f is continuous on the closed interval [a, b] and differentiable on the open interval (a, b) then there exists a number c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

What it means: Given two points a and b, the slope between those points will be attained as an instantaneous slope (ah, a derivative) by some point c that is between a and b.



In the picture, the slope from x = -1 to x = 2 is 1. I have drawn a green dashed line that shows another point (somewhere around x = 1.2) between -1 and 2 where the slope of the tangent line is also 1. There is also another place where the slope = 1. I think it is probably around x = -.5. Draw it yourself if you want

When to use it: To prove that the slope between to distinct points on the graph will equal the derivative of the function at some point x between a and b.

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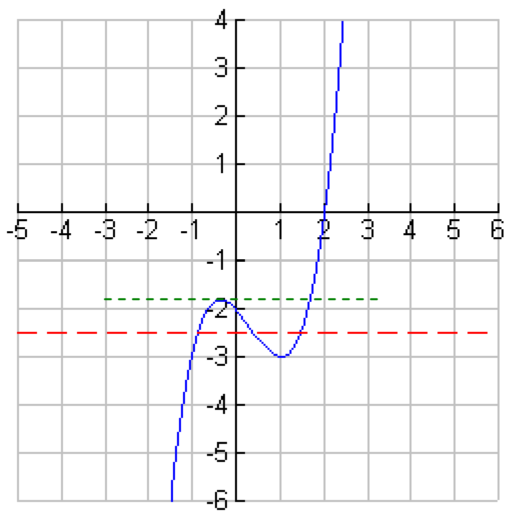
Rolle's Theorem

What it says: Let f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If f(a) = f(b) then there is at least one number c in (a, b) such that f'(c) = 0.



What it means: If a function has two places, a and b, where the y values are the same, then there will be a horizontal tangent somewhere between a and b.

In this picture, f(-.8546) = f(1.4516) = -2.5
The slope between x = -.8546 and x = 1.4516 = 0.
therefore, the slope at some point between will also be 0

This is used to prove MVT and can be thought of as a specific case of MVT – where the slope between the two points is 0.

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When to use it: Use it the same way as the MVT. You could also apply it to prove a theoretical max or min between two x values if you can't actually see the graph.