Algebra 3/Trigonometry
Final Exam Review

A. Solve for x:

1. $\log_{25} x = -\frac{1}{2}$
2. $\log_8 \frac{1}{32} = x$
3. $\log_3 x + \log_3(2x+51) = \log_4 256$
4. $\log_{243} \sqrt[3]{81} = x$
5. $12^{\log_{12} 2} = x^2$
6. $4(\log_{81} x)^2 + 3(\log_{81} x) - 1 = 0$
7. $\log_x\left(\frac{1}{8}\right) = 3$
8. $\sqrt{6}^{2x} = \left(\frac{1}{216}\right)^{2x-1}$
9. $\log_{25}\left(\log_3\left(\log_2 x\right)\right) = 0$
10. $\log x - \log(2x-3) = 2$
11. $16^{\log_2 3}$
12. $2^{3x-2} = 3^{2x+1}$ (simplify as far as possible WITHOUT a calculator)

B. Given $f(x) = 2x-9$, $g(x) = \frac{1}{x^2}$, $h(x) = \frac{2x-3}{x+5}$, $m(x) = \frac{x-3}{x+1}$ and $n(x) = \sqrt{x-3}$ find

1. $f^{-1}(x)$
2. $m^{-1}(x)$
3. $g^{-1}(x)$
4. $n^{-1}(x)$
5. $g \circ f$
6. $g \circ n$
7. $m \circ h$
8. $f(g(n(4))$
9. $m(h(g(2))$

C. Coordinate Geometry

1. Write the following in standard form and graph the following completely

a) $x^2 - 6x + y^2 + 4y = 3$
b) $2x^2 - 4x + y = 3$
c) $9x^2 + 4y^2 = 36$
d) $4x^2 + 16x - 9y^2 + 18y = 29$
e) $2y^2 - x + 2y = 1$

3. Write the equation of the hyperbola that has foci at (4, 5) and (-4, 5) and has slopes of asymptotes of ± 2
4. Write the equation of the conic that has as major axis endpoints (-13, 7) and (-1, 7) and minor axis endpoints (-7, 10) and (-7, 4).
5. Find the equation of the conic that has width points (12, 3) and (12, 1) and opens left.
6. Find the equation of the conic that has major axis endpoints (2,0) and (4,0) and a focus at (0,0).

D. Trigonometry

1. Find each of the following:

a) $\sin\left(\frac{11\pi}{6}\right)$ b) $\sin\left(\frac{3\pi}{4}\right)$ c) $\tan(315°)$

d) $\sec(210°)$ e) $\cos(150°)$ f) $\cot\left(\frac{9\pi}{4}\right)$

g) $\sec^{-1}(-2)$ h) $\cos^{-1}(-1)$ i) $\cot^{-1}\left(\sqrt{3}\right)$

j) $\cos\left(\tan^{-1}2 + \sin^{-1}\frac{1}{3}\right)$ k) $\tan\left(\cot^{-1}3 - \tan^{-1}6\right)$

2. If $\cos x = -\frac{3}{4}$ and $\tan x < 0$, find the values of the other five trig functions.

3. Graph the following functions completely.

a) $f(x) = -4\cos(2x+\pi)+1$ b) $f(x) = 2\tan 3x - 3$

c) $f(x) = 3\sin\left(x - \frac{2\pi}{3}\right)+2$ d) $f(x) = -\cot\left(\frac{x}{4}\right)+1$

e) $f(x) = -4\csc(2x+\pi)+1$ f) $f(x) = \frac{1}{2}\sec\left(x+\frac{\pi}{4}\right)+\frac{3}{2}$

4. Prove the identity $\frac{\cos^3 x - \sin^3 x}{1-2\sin^2 x} = \frac{\sec x + \sin x}{\tan x + 1}$

5. If $\sin\alpha = \frac{2}{5}$, α in quadrant 2 and $\sec\beta = 4$, β in quadrant 1 find the following.

a) $\sin 2\alpha$ b) $\cos(\alpha+\beta), \sin(\alpha+\beta)$ and quadrant of $(\alpha+\beta)$

c) $\cos\frac{\beta}{2}$ d) $\tan(\alpha-\beta)$ e) $\cot\frac{\alpha}{2}$

6. Solve the following for $0 \le x < 2\pi$

a) $\sin 2x - \tan x = 0$ b) $6 - 2\cos^2 x = 9\sin x$

c) $4(\sec 3x)^2 = 16$ d) $\cos^2 x \sin x = \cos x \sin^2 x$

Directions: Set up an equation and isolate the unknown completely. Do NOT use a calculator.

7. A triangular parcel of land is bordered on one side by 115 meters of hedges, and the other by flower beds with lengths of 76 meters and 92 meters. At what angle do the flower beds meet?

8. A triangular parcel of land is bordered on one side by hedges and the other by flower beds with lengths of 76 meters and 92 meters. If the flower beds meet at an angle of 47.0°, find the length of the hedge border.

Answers:

A.

1. $\frac{1}{5}$ 2. $-\frac{5}{3}$ 3. $\frac{3}{2}$ 4. $\frac{4}{15}$ 5. $\pm\sqrt{2}$ 6. 3, $\frac{1}{81}$ 7. $\frac{1}{2}$ 8. $\frac{3}{7}$

9. 8 10. $\frac{300}{199}$ 11. 3^4 12. $\frac{\log 12}{\log 8/9}$

B.

1. $\frac{x+9}{2}$ 2. $\frac{x+3}{1-x}$ 3. $\pm\sqrt{\frac{1}{x}}$ 4. x^2+3 5. $\frac{1}{(2x-9)^2}$ 6. $\frac{1}{x-3}$

7. $\frac{-x-18}{3x+2}$ 8. -7 9. $-\frac{73}{11}$

C.

1. a. $(x-3)^2+(y+2)^2=16$ b. $(x-1)^2=-\frac{1}{2}(y-5)$ c. $\frac{x^2}{4}+\frac{y^2}{9}=1$ d. $\frac{(x+2)^2}{9}-\frac{(y-1)^2}{4}=1$

e) $\left(y+\frac{1}{2}\right)^2=\frac{1}{2}\left(x+\frac{3}{2}\right)$ 3. $\frac{x^2}{16/5}-\frac{(y-5)^2}{64/5}=1$ 4. $\frac{(x+7)^2}{36}+\frac{(y-7)^2}{9}=1$

5. $(y-2)^2=-2\left(x-12\frac{1}{2}\right)$ 6. $\frac{(x-3)^2}{1}-\frac{y^2}{8}=1$

D.

1. a) $-\frac{1}{2}$ b) $\frac{\sqrt{2}}{2}$ c) -1 d) $-\frac{2\sqrt{3}}{3}$ e) $-\frac{\sqrt{3}}{2}$ f) 1

g) $\frac{2\pi}{3}$ h) π i) $\frac{\pi}{6}$ j) $\frac{2\sqrt{10}-2\sqrt{5}}{15}$ k) $-\frac{17}{9}$

2. $\sin=\frac{\sqrt{7}}{4}$ $\sec=-\frac{4}{3}$ $\csc=\frac{4\sqrt{7}}{7}$ $\tan=-\frac{\sqrt{7}}{3}$ $\cot=-\frac{3\sqrt{7}}{7}$

3. a) Start: $-\frac{\pi}{2}$, End: $\frac{\pi}{2}$; graph in class b) Asympt: $-\frac{\pi}{6},\frac{\pi}{6},\frac{\pi}{2}$; graph in class

c) Start: $\frac{2\pi}{3}$, End: $\frac{8\pi}{3}$; graph in class d) Asympt: 0, 4π, 8π; graph in class

e) Asympt: $-\frac{\pi}{2}$, 0, $\frac{\pi}{2}$; graph in class f) Asympt: $-\frac{3\pi}{4}$, $\frac{\pi}{4}$, $\frac{5\pi}{4}$; graph in class

4. (proof)

5. a) $-\frac{4\sqrt{21}}{25}$ b) $\frac{-\sqrt{21}-2\sqrt{15}}{20}$, $\frac{2-3\sqrt{35}}{20}$, QIII c) $\frac{\sqrt{10}}{4}$ d) $\frac{32\sqrt{21}+25\sqrt{15}}{39}$ e) $\frac{5-\sqrt{21}}{2}$

6. a) $0,\pi,\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4},2\pi$ b) $\frac{\pi}{6},\frac{5\pi}{6}$ c) $\frac{\pi}{9},\frac{2\pi}{9},\frac{4\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9},\frac{8\pi}{9},\frac{10\pi}{9},\frac{11\pi}{9},\frac{13\pi}{9},\frac{14\pi}{9},\frac{16\pi}{9},\frac{17\pi}{9}$

d) $0,\frac{\pi}{2},\frac{3\pi}{2},\pi,\frac{\pi}{4},\frac{5\pi}{4}$

7. $\cos^{-1}\left(\frac{115^2-76^2-92^2}{-2(76)(92)}\right)$ 8. $\sqrt{76^2+92^2-2(76)(92)\cos 47}$ 9. $x=\frac{35\sin 23}{\sin 63}$