

Limits

Lecture 14

$$(1) \lim_{n \rightarrow \infty} \left(\frac{7n + 3}{3n - 2} \right)$$

$$(2) \lim_{n \rightarrow \infty} \left(\frac{2n^2 + 5n - 2}{-3n^2 + 4n + 5} \right)$$

$$(3) \lim_{n \rightarrow \infty} \left(\frac{9n^2 - 5n + 3}{2n^3 + 7} \right)$$

$$(4) \lim_{n \rightarrow \infty} \left(\frac{5n^4 + 2n}{16n^3 - 9n} \right)$$

$$(5) \lim_{n \rightarrow \infty} \left(\frac{5n - 7n^4}{4n^4 + 3n^2 - 2} \right)$$

$$(6) \lim_{n \rightarrow \infty} \left(\frac{2n - 3}{5n + 4} + \frac{3n - 2}{2n + 1} \right)$$

$$(7) \lim_{n \rightarrow \infty} \left(\frac{5n - 1}{3n + 2} - \frac{n + 6}{2n + 7} \right)$$

$$(8) \lim_{n \rightarrow \infty} \left(\frac{n^3 + 1}{n^2 - 5} + \frac{2n + 3}{n^2 + 7} \right)$$

$$(9) \lim_{n \rightarrow \infty} \left(\frac{n^3 + 1}{n^2 - 5} \cdot \frac{2n + 3}{n^2 + 7} \right)$$

$$(10) \lim_{n \rightarrow \infty} \left(6 + \frac{(2n - 7)^2}{(3n + 5)^2} \right)$$

$$(11) \lim_{n \rightarrow \infty} \left(\frac{(1 - 3n)^3}{(5n^2 + 7)(4n + 3)} \right)$$

$$(12) \lim_{n \rightarrow \infty} \left(\frac{(5n - 3)^3}{(n^2 + 7)^2} \right)$$

$$(13) \lim_{n \rightarrow \infty} \left((-1)^n \cdot \frac{2n + 3}{3n + 5} \right)$$

$$(14) \lim_{n \rightarrow \infty} \left((-1)^n \cdot \frac{2n + 3}{5n^2 + 7} \right)$$

$$(15) \lim_{n \rightarrow \infty} \left(\frac{2n^2 + 1}{n - 2} - \frac{2n^2 + 3n}{n + 1} \right)$$

$$(16) \lim_{n \rightarrow \infty} \left(\frac{3}{n^2} - \frac{5}{n} \right) \left(6n + \frac{4}{n^3} \right)$$

$$(17) \lim_{n \rightarrow \infty} \left(\frac{2}{3} \right)^n$$

$$(18) \lim_{n \rightarrow \infty} \left(-\frac{4}{3} \right)^n$$

$$(19) \lim_{n \rightarrow \infty} \left(\frac{n!}{n^{10}} \right)$$

$$(20) \lim_{n \rightarrow \infty} \left(\frac{x^2}{2x^2} \right)$$

Infinite Geometric Series

Lecture 15

(1) Find the sum of the infinite geometric series for each of the following.

(a) $a = 7$, $r = \frac{1}{3}$

(b) $a = 11$, $r = \frac{\sqrt{3}}{6}$

(2) Write the first three terms of an infinite G.P. if $a = \frac{9}{2}$ and $S = \frac{15}{2}$.

(3) Find the range of values for which each of the following infinite geometric series converges.

(a) $1 + 3(x+1) + 9(x+1)^2 + 27(x+1)^3 + \dots$

(b) $1 + \frac{1}{8}(2x+3)^3 + \frac{1}{64}(2x+3)^6 + \frac{1}{512}(2x+3)^9 + \dots$

(4) Find r for the infinite geometric series in which $S = \frac{4+3\sqrt{2}}{2}$ and $a = 1 + \sqrt{2}$.

(5) Find all values of x for which each of the following infinite geometric series has the given sum.

(a) $\frac{2}{3} = 1 + 3x + 9x^2 + 27x^3 + \dots$

(b)

$\frac{2x+1}{2x} = 1 + x + x^2 + x^3 + \dots$

(6) Find the sum of an infinite geometric series if the sum of the first two terms is 36, and the third term is 3.

(7) Evaluate each of the following sums please.

(a) $\sum_{d=1}^5 12\left(\frac{1}{3}\right)^d$

(c) $\sum_{d=-1}^{\infty} 16\left(-\frac{1}{2}\right)^{d+1}$

(b) $\sum_{d=1}^{\infty} 12\left(\frac{1}{3}\right)^d$

(d) $\sum_{d=1}^{\infty} 12\left(\frac{\sqrt{5}}{2}\right)^{d-1}$

(8) Each side of a square measures 16. The midpoints of each side are joined to form an inscribed square. The midpoints of each side of this second square are joined to form a third square. If this process is continued endlessly, find the sum of the areas and perimeters of all the squares, including the original one.

Worksheet

1. Find the first three terms of an infinite GP whose sum is 8 if $r = \frac{1}{2}$.
2. Find the first three terms of an infinite GP whose sum is 8 if $r = -\frac{1}{2}$.
3. For what values of x will the sum of the following sequences exist?
 - a) $1 + 2x + 4x^2 + \dots$
 - b) $1 + \frac{1}{2}x + \frac{1}{4}x^2 + \dots$
 - c) $2 + 2(x - 3) + 2(x - 3)^2 + \dots$
4. For what value of x will the given sequence have the given sum?
 - a) $10 = 1 + x + x^2 + \dots$
 - b) $10 = 1 - x + x^2 - \dots$
 - c) $10 = 6 + 6(x - 2) + 6(x - 2)^2 + \dots$
 - d) $\frac{2x + 3}{2x} = 2 + 2x + 2x^2 + \dots$
5. The sides of a square are 24 inches. The midpoints of its sides are joined to form an inscribed square. If this process is continued without end, find the sum of the perimeters of all the squares.

Answers:

1. 4, 2, 1
2. 12, -6, 3
3. a) $-\frac{1}{2} < x < \frac{1}{2}$
- b) $-2 < x < 2$
- c) $2 < x < 4$
4. a) $\frac{9}{10}$
- b) $-\frac{9}{10}$
- c) 2.4
- d) $\frac{1}{2}$ (-3 is no good – why?)
5. $96(2 + \sqrt{2})$