# AREAS OF POLYGONS

SQUARE	 	
RECTANGLE		
PARALLELOGRAM		
TRIANGLE		
EQUILATERAL TRIANGLE	 	
RHOMBUS	 	
TRAPEZOID	 	
REGULAR POLY		
CIRCLE		
ARC LENGTH		
SECTOR		
SLIVER	 	
RATIO OF AREAS		
SAME BASE		
SAME HEIGHT		

#### Geo 9 Ch 11 **11.1 Areas of Polygons**

**Postulate 17** The area of a square is the square of the length of a side.



**Postulate 18** If two figures are  $\cong$ , then they \_\_\_\_\_

**Postulate 19** Area Addition Postulate : The area of a region is the sum of the areas of its non-overlapping parts.

Theorem 11-1 The area of a rectangle equals the product of the base and the height



Classify as true or false

- 1. If two figures have the same area, then they must be congruent.
- 2. If two figures have the same perimeter, then they must have the same area.
- 3. If two figures are congruent, then they must have the same area.
- 4. Every square is a rectangle.
- 5. Every rectangle is a square.
- 6. The base of a rectangle can be any side of the rectangle.

Given the following information for a rectangle, find the missing piece:

- 7. base = 12 m, height = 3 m, Area = \_\_\_\_\_
- 8. base = 9 m, height = \_\_\_\_\_, Area = 54 m<sup>2</sup>.
- 9. base =  $4\sqrt{3}$  m, height =  $5\sqrt{3}$  m, Area = \_\_\_\_\_
- 10. base = y-2, height = y, Area = \_\_\_\_\_

Find the area of the following, given consecutive sides of the figures are perpendicular







### 11.2 Areas of parallelograms, triangles, and rhombuses

**Theorem 11-2:** The area of a parallelogram equals the product of a base times the height to that base.



Look familiar ?? Watch this:





A parallelogram

surprise!! A rectangle

**Theorem 11-3** The area of a triangle equals half the product of a base and the height to that base.

chop and move



Theorem 11-4 The area of a rhombus equals \_\_\_\_\_\_



**11.2** Find the area of each figure. Make sure to draw a picture!!

- 1. Triangle with base = 6 cm and corresponding height = 9 cm.
- 2. Parallelogram with base  $4\sqrt{3}$  and corresponding height  $3\sqrt{21}$ .
- 3. Rhombus with diagonals 6 and 8.

4. An isosceles triangle with sides 12, 12 and 10.



5. An equilateral triangle with all sides 9.









Geo 9 Ch 11 **11.2** 



Area \_\_\_\_\_

10. parallelogram



Area \_\_\_\_\_



Perimeter = 48

Area = \_\_\_\_\_









Area \_\_\_\_\_

#### 11.3 Areas of trapezoids

What was a trapezoid? \_\_\_\_\_

What was a median?

**Theorem 11-5** The area of a trapezoid equals half the product of the height and the sum of the bases.

$$\mathbf{A} = \frac{1}{2}\mathbf{h} \left(\mathbf{b}_1 + \mathbf{b}_2\right)$$

Think: What is  $\frac{1}{2}(b_1 + b_2)$ ? So, another form of the equation would be

- A =
- 1. In trapezoid TRAP, RA || TP m $\angle$ T = 60, RA = 10, TR = 8 and TP = 15. Find the area.



2. In isosceles trapezoid ABCD, the legs are 8 and the bases are 6 and 14. Find the area.

3. A trapezoid has an area of 75 and a height of 5. How long is the median?

Find the area of each trapezoid.











Area \_\_\_\_\_

Area \_\_\_\_\_



#### 11.4 Areas of regular polygons

Each polygon can be thought of as a group of regular, congruent triangles. First, some definitions

**Center** : is the center of the circumscribed circle.

**Radius** : distance from the center to a vertex

Central Angle: an angle formed by two radii drawn to the consecutive vertices.

Apothem : the (perpendicular) distance from the center of the polygon to a side.

Identify each in the following picture:



**Theorem 11-6** The area of a regular polygon is equal to half the product of the apothem and the perimeter.

$$\mathbf{A} = \frac{1}{2} \mathbf{a} \, \mathbf{p}$$

Find the measure of the central angle of each figure:

1. A square

2. A regular hexagon





3. A regular octagon

4. 12-sided polygon



Find the perimeter and the areas of each figure: (umm...draw a picture?)

5. A regular hexagon with apothem 6

6. A regular octagon with side 5 and apothem 3.

7. An equilateral triangle with apothem 8.

Complete the table for the figure

8. A square:

	r	а	Р	А
a)	5√2			
b)		$\sqrt{3}$		



# 9. An equilateral triangle

	r	а	Р	А
a)	8			
b)			6√3	





# 10. A regular hexagon





## REVIEW Ch 11.1-11.4 AREA

- 1) Find The area of each of the following:
  - a) A rectangle whose base is 6 and whose diagonal is 10.
  - b) A square whose diagonal is 8.
  - c) A parallelogram having two adjacent sides of 12 and 15 cm with an included angle of  $60^{\circ}$ .
- 2) Find the dimensions of each of the following:
  - a) A square whose area is 144 cm<sup>2</sup>.
  - b) A rectangle whose area is 75 and whose base and height are in the ratio of 3:1.
  - c) A rectangle having an area of 135 and whose base is x+2 and height is 2x+1.

Find the areas of the following figures:



12) The area of an equilateral triangle is given. Find the length of a side.

a)  $16\sqrt{3}$  b)  $11\sqrt{3}$ 

- 13) Find the length of the shorter diagonal of a rhombus if
  - a) The length of the longer diagonal is 15 and the area is 90.
  - b) The lengths of the diagonals are in the ratio of 2:3 and the area of the rhombus is 147.
- 14) Find the length of an altitude of a trapezoid if
  - a) Its area is 72 and the sum of the bases is 36.
  - b) Its area is 80 and its median is 16.
  - c) The sum of the bases is one-third of the area of the trapezoid.
- 15) Find the area of a rhombus if its perimeter is 68 and one diagonal is 16.
- 16) Find the area of a triangle if the length of a pair of adjacent sides are 6 and 14, and the included angle is
  - a) 90° b) 30° c) 120°

- 17) Find the measure of a central angle for a regular octagon.
- 18) If the measure of an interior angle of a regular polygon is 150°, find the measure of the central angle.
- 19) Find the area of a regular hexagon inscribed in a circle having a diameter of 20 cm.
- 20) Fill in the table for a regular polygon having four sides :

	Side	Radius	Apothem	Area
a	6cm			
<u>b</u>				<b>49cm</b> <sup>2</sup>
C			5cm	
<u>d</u>		8cm		

21) The length of a side of a regular decagon is 20 cm. Find the apothem. Use trig.

Find the area of the following:

- 22) A square with a diagonal of  $\sqrt{6}$ .
- 23) A rectangle with base 12 and perimeter 38.
- 24) A triangle with sides of lengths 17,30, 17.
- 25) An equilateral triangle with height  $4\sqrt{3}$ .
- 26) A square with radius  $3\sqrt{2}$ .
- 27) A regular hexagon with perimeter 36.



31) A parallelogram has sides of lengths 12 and 18. The longer altitude has length 9. How long is the shorter altitude?

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#### **CHAPTER ELEVEN REVIEW ANSWERS**

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1) 48, 32, 90\sqrt{3}
2) 12 x 12, 5 x 15 9 x 15
3) 60
4) 35\sqrt{3}
5) 10√2

 9√3

7) 104
8) 52
9) 17.5
10) 162
11) 54 + 36\sqrt{3}
12) 8, 2√11
13) 12, 14
14) 4, 5, 6
15) 240
16) 42,21, 21\sqrt{3}
17) 45
18) 30
19) 150\sqrt{3}
20)
21) 30.8
22) 3
23) 84
24) 120
25) 16√3
26) 36
27) 54√3
28) 198
29) 24√2
30) 13
31) 6
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(7) Find the area of the isosceles triangle whose legs measure 8 and whose base is 12.

(8) Find the area and perimeter of an equilateral triangle whose height is  $4\sqrt{3}$ .

(9) Find the radius and area of a regular hexagon whose apothem is 4.

(10) Find the radius, apothem and area of a square whose perimeter is 24.





#### Answers

(1) area = 99(2) area = 12 , perimeter =  $8\sqrt{3}$ (3) area = 60(4) area = 50(5)  $a \triangle ABC = 24$ , AB = 10,  $CD = \frac{24}{5}$ (6) area =  $9\sqrt{3}$ (7) area =  $12\sqrt{7}$ (8) area =  $16\sqrt{3}$ , perimeter = 24 (9) radius =  $\frac{8\sqrt{3}}{3}$ , area =  $32\sqrt{3}$ (10) radius =  $3\sqrt{2}$ , apothem = 3, area = 36 (11) area = 80 (12) area =  $36\sqrt{3}$ , perimeter =  $24 + 12\sqrt{3}$ (13)  $a\Box ABCD = 96$ , DE = 12,  $DF = 2\sqrt{7}$ (14) area = 48 (15) area =  $40 + 16\sqrt{3}$ , perimeter =  $8 + 4\sqrt{6} + 18\sqrt{2}$ (16) area = 156 (17) area = 120, perimeter = 52, BE =  $\frac{120}{13}$ (18) area =  $25 + 50\sqrt{2}$ 

#### 11-5 Circumferences and Areas of Circles

Circumference of a circle with radius r  $\rightarrow$  \_\_\_\_\_\_

Circumference of a circie with diameter d  $\rightarrow$ 

Area of a circle with radius r  $\rightarrow$ 

The radius of  $\odot O$  is three times the radius of  $\odot R$ .

1. What is the ratio of the circumference to the diameter for each of the two circles?

- 2. Compare the circumferences of those same two circles.
- 3. Compare the areas of the same two circles.

4. Fill out the following chart for ⊙M

	r	D	С	А
a)	15			
b)		8		
c)			26π	
d)				100π

6. The diameter of the world's largest pizza is 16 m. Find the circumference of the crust.

### Geo 9 Ch 11 11.6 Cooperative Learning Experiment

#### Create your own formulas!!

Using the following problems as examples, create a general formula that you can use for any similar problem.

- 1) Find the area of the shaded region, called a sector, if the measure of the arc  $AB = 60^{\circ}$ , and the radius OB = 5.
- 2) Repeat #1, but  $AB = 135^{\circ}$  and OB = 4.
- 3) Repeat #1, but ACB =  $235^{\circ}$  and OB = 3.
- 4) Repeat #1, but AB = x° and OB = r. Share this with Mrs. McGrath. When this is correct, you will have the <u>formula for finding the area of a sector.</u>

Now repeat examples #1 thru #3, but instead of finding the area of the sector, find <u>the length of the arc</u>, given the same information.

- 5)  $AB = 60^{\circ}$ , OB = 5
- 6)  $AB = 135^{\circ}$ , OB = 4
- 7)  $AB = 235^{\circ}$ , OB = 3
- 8) Formula for finding the arc length (not measure, which is in degrees) on a circle. Use  $AB = x^\circ$ , and OB = r. Share this with Mrs. McGrath.





#### 11-6 Arc Lengths and Areas of Sectors

=

Sector of a circle is \_\_\_\_\_

The length of a sector is its portion of the whole circumference, and the area of a sector is its portion of the whole area. Therefore the formulas are:

Arc Length

Area of sector AOB =



Now find the area of the region bounded by AB and AB. We call it a **"sliver**". What were the steps taken to get this area? Write a basic **strategy** in words, then share it with Mrs. McGrath.



Next find the area between the lines. Set up a strategy first and share it with Mrs. McGrath.



Finally, do problem #26 on page 455. Again, set up a strategy before completing the problem.



#### 11-7 Ratios of Areas

# We will be comparing areas of figures by comparing ratios





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Theorem 11-7 If the scale factor of two similar figures is a:b, then

1) The ratio of the perimeters = \_\_\_\_\_

- 2) The ratio of the areas = \_
- 3. The ratio of the corresponding heights of two similar triangles is 3:5.
- a) The ratio of the corresponding sides is \_\_\_\_\_
- b) The ratio of the perimeters is \_\_\_\_\_
- c) The ratio of the areas is \_\_\_\_\_
- 4. The diameters of two circles are 10 cm and 9 cm. What is the ratio of their circumferences? Their areas?

\_\_\_\_\_

5. A pentagon with sides 5, 7, 8, 9, and 11 has an area of 96. Find the perimeter of a similar pentagon whose area is 24.

6. Find the ratio of the areas of triangles I and II, and triangles I and III.



/PQRS, T is a midpoint.

a) If the area of PQRS is 60, find the areas of the regions I,II,III,IV.





Triangle ABC is a right triangle, E and F are midpoints, D, H, and G divide AC into 4 = segments.

Find: a) the ratio of areas of  $\blacktriangle$  ABC to  $\blacktriangle$  EBF. the ratio of areas of  $\blacktriangle$  ABC to  $\blacktriangle$  GFC. b)  $\blacktriangle$  ADE to  $\blacktriangle$  GFC. the ratio of areas of c) the ratio of areas of  $\square$  DEFG to  $\blacktriangle$  ABC. d) e) the perimeter of DEFG if AC = 20А D Η E G В C F

(2) Find the area between a circle and an inscribed equilateral triangle if each side of the triangle measures 6.

(3) Find the area of a circle below if each side of the hexagon is 6.





(5) In a circle whose radius is 6, a sector has an area of  $15\pi$ . Find the length of the arc of the sector.

- (6) The lengths of the sides of a triangle are 4, 5, and 7. The shortest side of a similar triangle is 10. Find the perimeter of the second triangle.
- (7) Find the area of the shaded region in the figure below.



(8) Find the area of the shaded region in the figure below.



(9) Find the **sum** of the areas of the

shaded regions in the figure below.







(13) Find the area of the shaded region if□ABCD is a square, and arcs AED and DEC are semicircles.



(10) Find the area of the shaded region in the square below if D is the center of AC.



(12) Find the sum of the areas of the shaded regions if □ABCD is an isosceles trapezoid.



(14) Find the area of the shaded region and the length of arc AB in the figure below.





(19) Two similar triangles have areas of 12 and 27. The shortest side of the smaller triangle is 6. Find the shortest side of the larger triangle.

#### Ch 11 Sector Answers

(1) area =  $50\pi$ , circumference =  $10\sqrt{2\pi}$ (2) area between =  $12\pi - 9\sqrt{3}$ **(3)** 27π (4) arc length =  $\frac{8\sqrt{3}\pi}{3}$ , area shaded region =  $16\pi - 12\sqrt{3}$ **(5)** 5π **(6)** 40 (7)  $4\pi - 8$ **(8)** 4π (9)  $25\pi - 48$ (10)  $64 - 16\pi$ (11)  $9\pi + 18$ (12)  $24\sqrt{2} - 8\pi$ (13)  $48 - 8\pi$ (14) area =  $24\pi$ , arc length =  $6\pi$ (15)  $30 - 4\pi$ (16)  $16\sqrt{3} - 8\pi$ (17) (a)  $\frac{a \triangle ADC}{a \triangle BDC} = \frac{1}{1}$  (b)  $\frac{a \triangle I}{a \triangle II} = \frac{3}{5}$  (c)  $\frac{a \triangle II}{a \triangle III} = \frac{1}{1}$ (d)  $\frac{a \triangle I}{a \triangle IV} = \frac{9}{25}$  (e)  $\frac{a \triangle III}{a \triangle IV} = \frac{3}{5}$  (f)  $\frac{a \triangle I}{a \square ABCD} = \frac{9}{64}$ 

(18)  $a \triangle ABF = 80$ ,  $a \triangle BCF = 30$ ,  $a \triangle CEF = 18$ ,  $a \triangle DEF = 30$ 

(19) 9

# **Chapter 11 Supplementary Problems**

### Prior to 11.1 (Areas of Rectangles)

1. The length of a *rectangle* is (3x-4) and the width is (2x+1). Find the perimeter and area of this rectangle.

2. A 9-by-12 rectangular picture is framed by a border of uniform width. Given that the combined area of the picture plus the frame is 180 square units, find the width of the border.

3. If the perimeter of a square is 36 what is the length of a diagonal of the square? If the area of the square is also 36, what is the length of a diagonal of the square?

4. A paper towel tube has a diameter of 4 inches and a height of 11 inches. If the tube were cut and unfolded to form a rectangle, what would the area of that rectangle be?

### Prior to 11.2 (Areas of Parallelograms, Triangles and Rhombuses)

- 5. Find the area of a triangle with sides 10, 10, and 6.
- 6. Sketch a rhombus with perpendicular diagonals labeled  $d_1$  and  $d_2$ . Find the area of the rhombus by using the triangles created by the diagonals and come up with a simplified equation for the area in terms of  $d_1$  and  $d_2$ .
- 7. Draw a parallelogram and drop an *altitude* .( The perpendicular distance between 2 parallel lines.) Can you create a rectangle from the parallelogram and if so what would the area be?



8. Draw an equilateral triangle. Drop an altitude to one of the sides. If a side of the triangle is labeled "s", come up with a formula for the area of an equilateral triangle using "s". Does this formula work for any triangle?

### Prior to 11.3 (Areas of Trapezoids)

- Sketch a diagram of an isosceles trapezoid whose sides have lengths 7 in., 10 in., 19 in., and 10 in. Find the **altitude** of this trapezoid (the distance that separates the parallel sides), then find the enclosed area.
- 10. A trapezoid has two 60 degree angles and 8 in. and 12 in. parallel sides. How long are the nonparallel sides? What is the trapezoid's area?



## Prior to 11.4 (Areas of Regular Polygons)

11. A hexagon is inscribed in a circle of radius 6. ( All regular polygons can be inscribed in a circle) Find the area of the hexagon by dividing it into shapes you know how to find the area of.



12. Find the area of a square inscribed in a circle with a radius of 6.



#### Prior to 11.5 (Circumferences and Areas of Circles)

13. The track around Parsons Field is shaped like a rectangle with a semicircle on each of the ends. The distance around the track is one-quarter mile. The straightaway is twice as long as the width of the field. What is the area of the field enclosed by the track to the nearest square foot? Use a calculator.



14. The figure shows three circular pipes all with 12 inch diameters that are held together by a metal strap. How long is the band?



#### Prior to 11.6 (Arc Lengths and Sector Areas)

- 15. In a group of 12 students, only 4 of them like olives on their pizza. If they are sharing a 16 in. pizza, what is the area of the part of the pizza covered with the olives?
- 16. A **sector** of the circle is the region bounded by two radii and an arc. It's **area** is a fractional amount of the area of the circle. Compare the *areas* of two *sectors* if:

a) they have the same central angle but the radius of one is twice as long as the radius of the other.

b) they have the same radius but the central angle of one is twice as large as the central angle of the other.



17. Draw a large triangle ABC and mark D on segment AC so that the ratio AD:DC is equal to 3:4. Mark any point P on segment BD.

- a) Find the ratio of the area of  $\triangle BAD$  to the area of  $\triangle BCD$ .
- b) Find the ratio of the area of  $\triangle PAD$  to the area of  $\triangle PCD$ .
- c) Find the ratio of the area of  $\triangle BAP$  to the area of  $\triangle BCP$ .



18. The areas of two similar triangles are 24 square cm. and 54 square cm. The smaller triangle has a 6 cm. side. How long is the corresponding side of the larger triangle?

19. The areas of two circles are in the ratio of 25:16. What is the ratio of their radii?